

A CREDIT CONDITIONS INDEX FOR NORWAY

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Preface

This thesis was submitted as a final part of the requirements for the degree *Master of Economic Theory and Econometrics* at the University of Oslo. It has been written while I enjoyed a student internship in the Macroeconomics Group at the Research Department of Statistics Norway during the fall of 2009 (part time) and the spring of 2010 (full time). I am grateful to all my colleagues for providing a great atmosphere at work.

The greatest acknowledgements are reserved for Eilev S. Jansen, senior researcher at Statistics Norway, and Ragnar Nymoen, Professor at the University of Oslo, who have been my supervisors for this thesis. Even though, as Dylan (1965) puts it, *you don't need a weatherman to know which way the wind blows*, you do need supervisors to tell you which way your thesis is going. Both have contributed with helpful advices, academic stimulation and general motivation. Jansen deserves special attention and gratitude for suggesting the particular topic for this thesis.

Thanks also to Sigbjørn Atle Berg and Gunnvald Grønvik who read through the extended version of Chapter 2 (Krogh, 2010, forthcoming). They provided valuable comments and necessary corrections.

Furthermore, I wish to thank Laila Haakonsen and Olav Stensrud at Statistics Norway and Vetle Hvidsten at Norges Bank for helping me track down the data I have used. Arild Eide Johansen and Øivind Jonassen at Finanstilsynet have helped me find documents (letters sent from Finanstilsynet to the financial institutions) in the archives of Finanstilsynet that are not available elsewhere.

I must also thank my office mate André Kallåk Anundsen. Our cooperation has been the source of several interesting and useful discussions.

Summary

Fernandez-Corugedo and Muellbauer (2006) represents a novel paper where a credit conditions index (CCI) is estimated for the UK. The CCI is intended to capture supply-side shifts that are due to structural changes and other shocks to the financial sector, controlling for e.g. changes in the interest rate or the level of output. This index will therefore allow researchers to control for the institutional development in their empirical work and take into account how some of the mechanisms in the credit market can have changed over time. The index of Fernandez-Corugedo and Muellbauer seems to capture the main developments in the UK including the extensive deregulation of the credit market during the 1980s. Succeeding works (e.g. Aron, Muellbauer, and Murphy (2008)) have proven that the including the CCI improves the performance of some econometric models.

If the methodology of credit condition indices is to gain increased relevance and usage more empirical evidence is needed. The main objective of this thesis is therefore to estimate a CCI for Norway, following the setup and method of Fernandez-Corugedo and Muellbauer (2006). To do so I go through several steps. Chapter 2 outlines how the regulation of Norwegian credit markets has developed since the 1970s. I have combined various sources in order to provide a relatively detailed summary of the deregulation process that took place. As far as I am aware of, there exists no other unified presentation of this kind, making this chapter important in its own respect. This also motivates that a more detailed version of Chapter 2 will be made available in Krogh (2010, forthcoming). In Chapter 3 I consider what tendencies in credit conditions that can be drawn from the mortgage survey conducted by Finanstilsynet. The purpose of Chapters 2-3 is to provide sound qualitative evidence for the structural development of the Norwegian credit markets since the 1970s. The requirement for a sensible estimate of the CCI must be that it is in line with this qualitative information.

Chapter 4 contains the model I will use to estimate the CCI and also sketch what I think the CCI will look like. I formulate a system of two equations to explain the development in secured and unsecured debt relative to income. Chapter 5 explains how the model is estimated using a maximum likelihood approach and it also provides some considerations of practical problems that may arise when the model is estimated. The end result of this chapter is a Stata command^a that is tailor-made to estimate the model. The description of the maximum likelihood framework applied and the derivations that accompany it can be useful for other researchers that want use their own maximum likelihood codes to estimate a nonlinear SUR model. This can be relevant if their model entails a variation that is not permitted by the standard commands that exist.

The estimates are presented in Chapter 6. I detect long-run relationships that have fairly

^aAll estimations have been performed with the statistical software Stata (StataCorp, 2007; Rel. 10).

reasonable coefficients, but with some exceptions. The implied CCI has a shape that matches most of the ex ante expectations that I have, given the evidence in Chapters 2-3, but both the CCI and the demographic variable seem to get too large coefficients in the equation for unsecured debt. I judge my results to represent a very useful first step, but I do think that the model probably ignores an important interaction between secured and unsecured debt that has led to a shift from the latter to the former type of debt. I describe how the weaknesses of my results point in this direction. Chapter 7 concludes.

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1 Introduction

The fact that financial markets matter, and that their apparent instability can lead to events that have grave consequences, is a recurrent issue in both economics as a field and in the public debate. The financial crisis, which took off after the collapse of Lehman Brothers September 15th 2008, gave the world a reminder of how dire the results might become when the situation gets out of hand. Norway has been relatively shielded from the current crisis, but the experiences from the banking crisis in the early 1990s are still fresh in memory.

The recent events will undoubtedly spur the interest for financial instability and initiate further research related to its consequences for the macroeconomy.¹ In this respect it is also relevant to investigate how financial markets have developed over time, in particular the effects of the extensive financial deregulation that has taken place in many European countries since the 1980s, including Norway. According to DeBelle (2004) the process of deregulation (which helped reduce the incidence of credit constraints) has been one of the main reasons for the increase in household borrowing in Western countries the last decades. However, this argument is only qualitative and it would be of great importance to have some notion of how strong *quantitative* effects one can attribute to financial reform and to deregulation. Furthermore, it has been argued by Goodhart, Hofmann, and Segoviano (2004) that the process of deregulation and also the creation of somewhat procyclical capital requirements have changed how the business cycle works and led to more prominent boom-bust cycles. If this is true, great interest would be attached to a measure of *when* the most important institutional changes took place. It is therefore a need for methods that can help us quantify the relative size of shocks to the credit markets and also provide us with some idea of when these shocks occurred.

One way of quantifying the effects of deregulation and other unobservable variables affecting the credit markets could be to assume that the impact of these factors can be summarized in one single credit conditions index (CCI). This index would, if successfully estimated, capture supply-side shifts such as those stemming from financial liberalization. Further, it would also partly capture shifts we would expect to occur in the midst of a financial crisis. The purpose of it, as I see it, is mainly to have an instrument that allows you to control for the historical development and hence improve your ability to explain events *ex post*.²

¹See Goodhart, Sunrind, and Tsomocos (2004) for an example of a work that acknowledged the challenge already prior to the crisis, and Blanchard, Dell'Ariccia, and Mauro (2010) for an example of how the "mainstream" has intentions of following.

²An alternative approach has been explored by the developers of various financial condition indices (FCIs). Hatzius, Hooper, Mishkin, Schoenholtz, and Watson (2010) describes seven FCIs that have been constructed by different institutions, among others Goldman Sachs, the Federal Reserve and the OECD. It also constructs a new FCI based on an unbalanced panel of 45 different financial indicators. The FCIs vary with respect to how they are constructed, but their common objective is to summarize the information about the *future* development of the economy contained in various financial variables, like various stock indices, interest rates and yield curves. This makes the FCIs a broader type of indices than the CCIs. As I interpret them, the value of an FCI at any point

Muellbauer and Murphy (1993) was one of the first attempts to estimate an index measuring the extent of financial liberalization. In this paper they consider annual UK data for the period of 1967 - 1990. Based on an assumption that lenders (or borrowers) aim for a constant debt-service to income ratio, they regress the loan to value ratio for first-time buyers on the log of the tax adjusted mortgage interest rate, the log of the house price to income and time dummies for all the years in the data set. The dummy-coefficients are set to zero in those years the degree of financial liberalization can be assumed to have remained constant, and the resulting financial liberalization index is basically the sum of all the dummies (times their estimated coefficient) plus a set of control variables. The index is then used to allow for a time-varying wealth effect in a consumption model.

Fernandez-Corugedo and Muellbauer (2006) develops the method from Muellbauer and Murphy (1993) further. They have a rich dataset which they use to construct 10 different indicators for the credit conditions in the UK. A 10 equation system is formulated and a wide range of controls are included as explanatory variables for the indicators. Then, to extract a CCI they assume that there is a common, but unobservable, variable affecting all the indicators. This variable is assumed to take the form of a "spline function" plus some policy variables. The spline function is a piecewise linear function that changes slope only in the first quarter of the year (for a more precise definition, see Chapter 4).

Two of the credit indicators used are data on (the log of) mortgage and non-mortgage household debt. In addition they have a comprehensive set of micro data with more than a million observations of mortgages for first-time buyers. They use this to find loan-to-value ratios (LVRs) and loan-to-income ratios (LIRs) for first-time buyers. They separate the data by age (less than 27 and 27 plus) and region (North and South) such that they end up with 8 indicators in addition to the two mentioned above.³

The authors include prior assumptions regarding the slope of the spline function in some of the years (based on qualitative information regarding the liberalization process in the UK) and also on the sign of other variables (based on theoretical considerations). The system is estimated as a nonlinear Seemingly Unrelated Regression (SUR) model. The priors are imposed sequentially such that coefficients violating their priors are set to zero (if several violates simultaneously, the one violating its restriction "the most" is first set to zero). The resulting CCI is illustrated in Figure 1.1. It has a reasonable shape and is in line with the institutional development presented in the paper.

A second work that contributes with CCI estimates is Blake and Muellbauer (2009), see

in time reflects how current financial shocks should affect future economic activity *ex ante*. This approach is therefore related, but clearly distinct from the CCI strategy. Future work should consider to clarify the connection between CCIs and FCIs.

³For the estimation purposes, they assume that both the LIR and LVR of each group are logistically distributed such that log-odds ratios for LIRs larger than 2.5 and LVRs larger than 0.9 can be constructed.

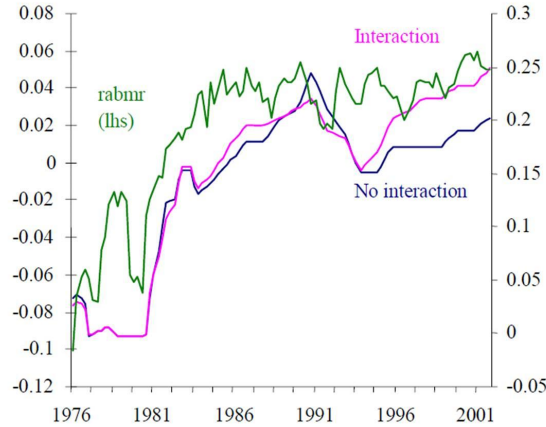


Figure 1.1: The CCI from Fernandez-Corugedo and Muellbauer (2006).

The line labelled "No interaction" is the standard CCI, that labelled "Interaction" is the result when the CCI is interacted with a risk measure. "rabmr" is the real mortgage interest rate.

also Oxford Economics (2009). This report gives estimates for house price and mortgage stock equations for several countries, and in that context also estimates of some form of credit conditions indices. For the UK they have the CCI of Fernandez-Corugedo and Muellbauer (2006), and they extend the index to 2008 using a "non-linear spline function" – the 4 quarter moving average of the spline function used by Fernandez-Corugedo and Muellbauer. The coefficients are estimated in a system of two equations; one equation for house prices and one for mortgage debt.

In addition, to make the UK results comparable with the results for the countries where no other indices are available, they estimate a new CCI for the UK based on this non-linear spline in the full sample. The resulting index tracks the pattern of the original CCI fairly well. This is a very interesting finding and it indicates that the "structural trend" detected by the framework of Fernandez-Corugedo and Muellbauer (2006) is quite robust. Similar two-equation systems are estimated for several other countries, including among others Germany, Spain and Italy, resulting in credit condition indices for these countries as well.

In his study of house prices in Australia, Williams (2009) uses the same methodology as Fernandez-Corugedo and Muellbauer (2006) to extract a variable for financial liberalization, but here the CCI is extracted from a single equation for housing prices. The CCI is constructed as a trend which is permitted to have break-points at 3 places. The positions of these break-points are guided by his estimates of a stochastic, unobserved trend from the STAMP software (Koopman, Harvey, Doornik, and Shephard, 2000). With only three break-points his CCI has a "simpler" shape than the UK index of Fernandez-Corugedo and Muellbauer (2006) but it fits well with the institutional background for Australia described in the paper.

An application that illustrates the usefulness of a credit conditions index is Aron, Muellbauer, and Murphy (2008). The strategy of this paper is to apply the CCI estimated by Fernandez-

Corugedo and Muellbauer (2006) in a consumption function. Changes in the credit conditions are argued to affect consumption in several ways. There is a collateral effect as illiquid wealth becomes more "spendable" when credit conditions are eased. Next there is an effect on the loan-to-value and loan-to-income ratios available for first-time buyers (allowing them to save less). Lastly there might also be changes in the interest rate response (access to credit gives better chances for inter-temporal smoothing). Hence the consumption function being estimated has both a direct effect from credit market liberalization and indirect effects through interaction with several of the variables.

When estimated on UK data, the introduction of the CCI as an extra variable improves the performance of the model. The CCI has an independently positive impact and by interacting it with other variables they find that it reduces the "pure" effects of housing wealth, expected income growth and the change in interest rates. The coefficients of the interaction terms get the same sign as their main coefficients. Hence, parts of the effects that in a model without a CCI would have been attributed to the variables alone, are simply due to changes in the underlying "credit regime".

If the methodology of credit condition indices is to gain increased relevance and usage more empirical evidence is needed. The main objective of this thesis is therefore to estimate a CCI for Norway, following the setup and method of Fernandez-Corugedo and Muellbauer (2006). To do so I go through several steps. Chapter 2 outlines how the regulation of Norwegian credit markets has developed since the 1970s. I have combined various sources in order to provide a relatively detailed summary of the deregulation process that took place. As far as I am aware of, there exists no other unified presentation of this kind, making this chapter important in its own respect. This also motivates that a more detailed version of Chapter 2 will be made available in Krogh (2010, forthcoming). In Chapter 3 I consider what tendencies in credit conditions that can be drawn from the mortgage survey conducted by Finanstilsynet. The purpose of Chapters 2-3 is to provide sound qualitative evidence for the structural development of the Norwegian credit markets since the 1970s. The requirement for a sensible estimate of the CCI must be that it is in line with this qualitative information.

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⁴All estimations have been performed with the statistical software Stata (StataCorp, 2007; Rel. 10).

model. This can be relevant if their model entails a variation that is not permitted by the standard commands that exist.

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2 Institutional background

The main sources for this chapter have been

- *The National Budget and the Revised National Budget from the Ministry of Finance: Finansdepartementet* (1969a-2007a)⁵ and *Finansdepartementet* (1970b-2008b)
- *The Annual Report from the Central Bank of Norway: Norges Bank* (1970a-2008a)
- *The Annual Report from the Financial Supervisory Authority of Norway: Finanstilsynet* (1986a-2008a)

Finansdepartementet (1970b-1978b) contains appendices discussing the credit policy of the government and has been the most important source for the period 1970-78. In Norges Bank (1970a-1986a) we find thorough descriptions of the credit policy and this has been used as the main source for the period 1978-1986. After 1986 these descriptions were omitted from the annual reports, but the reports still contain letters of announcement sent from Norges Bank to the financial institutions, and this is an important source of information (both before and after 1986). The description of the credit policy found in Finansdepartementet (1985a-2007a) is better than in previous editions of the National Budget (Finansdepartementet, 1969a-1984a) and is useful for our purposes. Furthermore, Finanstilsynet (1986a-2008a) is an important source for material on bank losses and market conditions from 1986 and onwards, and also letters sent from Finanstilsynet to the financial institutions are valuable.⁶ If a source different from these has been used, I will state that explicitly.

2.1 Status in the 1970s

The credit and monetary policy of the government had two main goals: A low and stable level of interest rates, and a careful injection of credit into the economy (see a contemporary description in Eide (1973)). The available credit was primarily to be used for investments, and the government wanted to limit households' borrowing for consumption purposes. The basis for credit policy during this period was the Credit Law of 1965. This law specified the instruments available for the authorities and it replaced earlier laws (see Norwegian Official Reports (1980) for a brief description of the law). The main goal of the new law was to change credit policy from being settled with explicit agreements between the government and the financial sector towards more indirect control where the authorities affected the liquidity of the credit market instead.

⁵Note that the national budget of year t is published in the autumn the year before.

⁶Most of these are downloadable at www.finanstilsynet.no. Letters that are not on the web are available in the archives of Finanstilsynet.

The main document for the credit policy was the credit budget, of which the government published a comprehensive version every year. The budget specified the amount of credit the authorities found it desirable for the financial institutions to supply in the course of the year, and also how this was to be shared between the different parts of the financial system.⁷ To make sure that the budget was met the government had several different regulations. In the early 1970s these can be described as follows:⁸

Quantitative regulations: *a)* Primary reserve requirements: Minimum requirement for the percentage of total assets that had to be held as primary reserves (i.e. deposits in Norges Bank, Postal Giro deposits and Post Office Savings Bank deposits (state banks), Treasury notes and notes and coins); *b)* Placement requirements: Requirement to invest a given percentage of the increase in total assets in the bond market; *c)* Additional reserve requirements: Requirement to put aside extra reserves given as a percentage of further growth in lending if lending exceeded a given limit; and *d)* Other direct regulations: The Credit Law also made it possible for the government to set discrete rules for permitted lending growth as well (i.e. less flexible for the firms but more direct control for the government) where a financial penalty was the result if a company exceeded its limit.

Interest rate controls: The most important interest rates on loans from the banks were dictated by the government through interest rate norms.

Foreign exchange controls: Transactions with and access to foreign exchange was extensively regulated (i.e. capital mobility was low). Any sale or purchase of foreign exchange had to go through one of the authorized foreign exchange banks. Banks' access to lend with foreign banks, as well as their currency holdings, were directly regulated. If businesses or private persons wanted to take up a loan abroad (or a foreign currency loan through a domestic bank), a licence from the authorities was needed. The number of licences was limited and these were in general only given if the purpose of the loan was investments or activities related to exports. For the oil and shipping sectors a licence was not needed. The private sector was not allowed to buy foreign securities and foreigners' access to buy Norwegian securities was very limited as well.⁹

⁷The budget was announced together with the National Budget every fall. In addition a revised credit budget was released together with the Revised National Budget in the spring.

⁸A part of the credit policy that I will not discuss is the existence of state banks. The government operated a total of 9 state banks that had been created to provide credit either to regions or to groups/causes that otherwise would have problems getting affordable credit in the private market like for instance students, fishermen and the agriculture sector. Most of these are still operated at present date. The most important state bank for ordinary households was one granting house mortgages (Husbanken), and it was a part of the house-building policy of the government. State banks were quite important as long as credit was rationed, but their role decreased some in relevance when the credit market was deregulated. See Norwegian Official Reports (1995) for a thorough report on the subject of state banks.

⁹Domestic residents could trade foreign securities on the so-called "switch market". This was not of great

Of the quantitative regulations, *a*) and *c*) were used to control the supply of credit from banks, while both banks and life insurance companies were affected by *b*). Through this the government secured a minimum demand for bonds in the bond market. Since the issuing of new bonds was regulated (not all firms could issue bonds), the supply of bonds was also controlled by the authorities. *d*) was used actively to limit lending from finance companies and non-life insurance companies.

Besides the regulation that was a part of the active credit policy, is also existed prudential regulation in the form of:

Capital requirements: Minimum requirement for the size of an institution's capital (share capital, reserve funds, retained earnings and subordinated debt) as a percentage of total liabilities (possibly deducted for the capital itself and some other assets – there were some changes over time).

Liquidity requirements: Minimum requirement for the size of an institution's liquid assets as a percentage of total liabilities. Liquid assets was defined basically in the same way as primary reserves. The main purpose was to secure that institutions were able to meet their short-term obligations.

Neither of these requirements were of any importance before the end of the 1980s. The primary reserve requirements normally secured that the liquidity requirements were met as well, and the capital requirement was seldom binding. The capital requirement is discussed more in Section 2.4.2.

A very short description of how policy was conducted in the period of 1970-1988 would be that the authorities continuously adjusted their instruments (the reserve requirements and other direct regulations) in an attempt to keep the growth of credit under control. This was relatively successful in the beginning of the 70s, but later the effectiveness declined steadily.

Primary reserve requirements were frequently adjusted in order to withdraw or provide liquidity for the banks. To illustrate how often they were changed, the primary reserve requirement for commercial banks is given in Figure 2.1.¹⁰ An increased primary reserve requirement had the effect of reducing the "lendable" amount of capital, given the level of reserves, making it more expensive to extend new loans.

While changing the primary reserve requirements was a part of the current operations in the conduct of credit policy, additional reserve requirements were only used when the situation got particularly "out of hand". An additional reserve requirement worked almost as a progressive primary requirement; a very high reserve requirement as soon as a specific limit for the growth

importance and it was closed in the middle of 1984.

¹⁰Savings banks were given a separate requirement which was usually a bit lower.

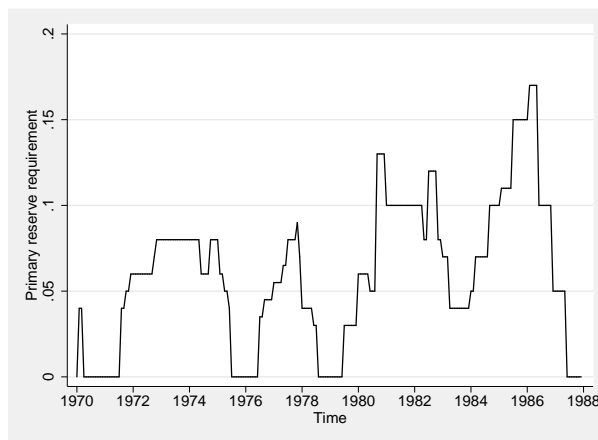


Figure 2.1: Primary reserve requirement for commercial banks, 1970-1987.^a

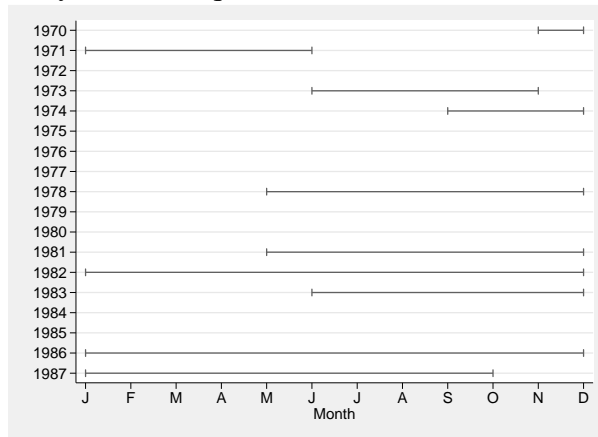


Figure 2.2: When additional reserve requirements were in place, 1970-1987.^b

^aRequirement for banks in "the south" (i.e. banks that did not have their main office in one of the northernmost counties, Nordland, Troms or Finnmark.). The series is based on monthly observations (if requirements were changed in the middle of a month, it is graphed as constant at the new level through the entire month). In the end of 1974 and through 1975 there was a lowering of the requirement for small banks. The basis of calculation was updated in 1978, 1982, 1985 and 1986. The first three times this worked as minor easing of requirements, while it was a small tightening in 1986.

^bThe lines indicate the periods of which a requirement was active (from when it was made public until expiration). The lines should be interpreted as follows: If a line ends in December and another line continues in January the next year, they represent the same requirement. Those in 1970, 1973 and 1974 were only for commercial banks. That of 1978, 1981 (until March 1982) and that of 1986-1987 were for both types, while that of 1982 (from March) was only for savings banks. There was no "standard" requirement: most incidents differed in some ways. Some requirements were also updated during their duration.

in lending was exceeded. Figure 2.2 shows when additional reserve requirements were active until the last was removed in October 1987.

In addition to the reserve requirements that limited lending from the banks, finance companies and non-life insurance companies were in almost every year from 1971 to 1988 (with a few exceptions) regulated directly. This meant that the companies were given a percentage limit for how much their lending could increase. The authorities could also adjust the placement requirement as a policy instrument but this was not used very actively.

Figure 2.3 illustrates how the operative difficulties of "credit planning" became larger over time. This figure gives the percentage deviation between the credit supplied during a year relative to what the politicians had put up as bounds in the credit budget. It indicates how the authorities' grip with the credit market became looser throughout the period, both due to a process of financial innovation (institutions were doing their best to sidestep the rules), change in efforts made to keep the flow of credit within the bounds of the budget (was the government actually expecting the credit budget to be met in the mid 1980s?) and the gradual deregulation that took place.

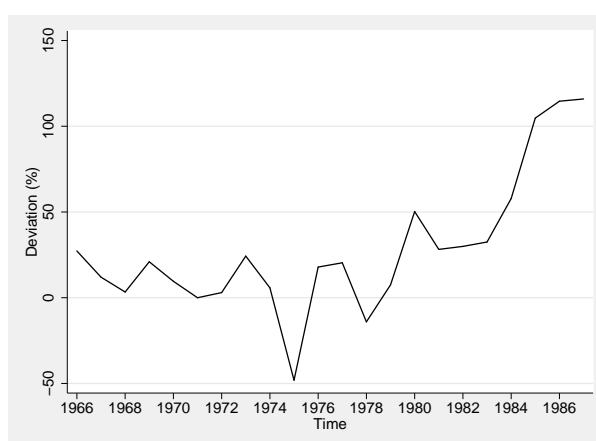


Figure 2.3: Percentage deviation between the actual credit supplied and the bounds of the credit budget, 1966-1987 (source: Norwegian Official Reports (1989)).

There are two sharp drops in Figure 2.3: one in 1975 and one in 1978. In 1975 it was a downturn in the Norwegian economy, and a part of the counter-cyclical measures was an expansionary credit policy. Banks were encouraged to provide credit for export companies such that they could increase their inventories as the downturn was partly driven by a contracted world economy. Various regulations were temporarily eased and the permitted level of lending from the state banks and the size of the credit budget were both increased. However, as seen in Figure 2.3, lending grew only slightly and the total credit supply ended far below the limits of the budget.

The dip in 1978 coincided with a *contractionary* credit policy.¹¹ Lending ceilings for both state banks and private banks were reduced. In addition the government made the private banks agree to limit the amount that households borrowed for consumption purposes.¹² During the same period there were also changes in the interest rate and foreign exchange controls: see Section 2.3. Of these, the change in the interest rate controls probably had the most rapid effect and it led to an increase in the interest rate level. In September 1978 a price and wage freeze was introduced, a sign of how drastic the situation was this year. The freeze lasted until the end of 1979.¹³

2.2 The grey market

Without doubt the authorities understood that the more regulated the ordinary credit market was, the stronger were the incentives for agents to enter the unregulated credit market (the "grey market").

Since the credit regulations were concerned with regulating financial *institutions*, it existed an opening for an unregulated market with direct lending between lenders and borrowers, usually using a finance broker as an intermediary. Even though we do not have complete statistics for the size of the grey market, we know that it started to grow during the late 1970s, and became considerable in the early 1980s. The existence of a relatively well-functioning grey market clearly reduced the effect of the credit regulations. If the government attempted to reduce the growth in lending from banks, borrowers and lenders could meet in the unregulated market instead. Financial institutions could also enter this market with loans outside their balance sheets. The banks' involvement in this market was also to put up guarantees for the loans, making a loan in the grey market less risky.

There exists data on the volume of the guarantees issued by the financial institutions, and also loans brokered by finance brokers without guarantees (but on this it is incomplete coverage). Based on these data the grey market had a very modest size in the mid 70s, but it grew rapidly starting in 1979 (see Norwegian Official Reports (1983, Table 10A)). By the end of 1981 the guarantees plus loans brokered was at about 11 billion kroner, by 1983 20 billion and by the 3rd quarter of 1987 it peaked (in nominal terms) at 34 billion kroner. At this point in time

¹¹This policy was intended to reduce private consumption growth and lower wage and price inflation in order to improve the competitiveness of the export industries.

¹²The deal was to reduce this kind of loans by 2,000 mill. kroner during 1978. It was made effective by the fact that Norges Bank required the agreement to be followed if a bank was to get access to the automatic lending from the central bank. A similar deal was agreed upon in 1979 as well.

¹³A further tightening of the credit policy came in 1979 when A- and B-loans were introduced as the new system of central bank loans to the banks. A-loans were quite ordinary central bank loans, but B-loans were loans that came with very strict conditions almost mimicking additional reserve requirements. If a bank's quota for A-loans was spent, B-loans was the only alternative. This system gave another instrument to limit the growth in lending. However, it was not as effective as one hoped for and the system with B-loans was suspended from July 1983.

that amounted to roughly 5% the size of the total assets of commercial and savings banks (see tables found in Norges Bank (1983b-1989b)). The real size of the market was probably even larger. We should therefore keep the grey market in mind when evaluating the effects of the credit regulations, especially in the 1980s. Its presence reduced the reliability of the available statistics and it became, by reducing the effectiveness of the regulations in place, an important motivation for the deregulation process.

2.3 The deregulation process

Let us now go through how the credit market was deregulated, considering each part of the credit market separately.

2.3.1 Removal of interest rate controls

The interest rate norms were withdrawn as early as in December 1977 (except for some types of mortgage loans). This entailed that banks were permitted to freely charge any interest rate. The authorities wanted to generate a general increase in the interest rate level, such that the real interest rate would turn positive,¹⁴ curbing total demand for credit. Furthermore, the control of the interest rates on the bond market was relaxed considerably (but not the control of whom that could issue).

However, when the prize freeze was introduced in 1978 this also covered interest rates, such that these were under government control again. In December 1979 the freeze ended, but the control of the interest rates was kept despite previous intentions of allowing it to float freely. The interest rate policy was re-formalized into the system of interest rate declarations the fall of 1980, a system where the Minister of Finance occasionally issued declarations with a ceiling for what it would accept for various interest rates. This was similar to the interest rate norms but less strict.

The interest rate declarations were abandoned from September 1985 and after this the interest rates floated freely.

2.3.2 Liberalization of the bond market

The liberalization of the bond market followed a pattern similar to that of the interest rate. First liberalization, then a period of re-regulation, and then gradual liberalization again. From the 1st of October 1980, the regulation of bond issuing was liberalized, in principle a complete liberalization of the supply side of bonds and private businesses as well as loan associations were

¹⁴The real interest rate had been negative most of the 70s. As one saw that net debt increased with income, this also had an undesirable distributional effect.

free to issue as many bonds as they liked. The demand for bonds continued to be stimulated by the placement requirement, which was kept unchanged.

The effect of the deregulation was, not surprisingly, a large increase in the flow of credit in the bond market, and it was stronger than what the government had envisaged. Therefore, in October 1981, the government saw it necessary to reintroduce some regulations in the bond market. Loan associations, who had been responsible of about half of the bonds issued from the private sector and municipalities the last 12 months, were denied to issue any new bonds the rest of the year. However, businesses could continue to issue bonds, making the market still more liberal than pre-1980. New regulations for the bond market were presented in March 1982, but these were far from as strict as those that existed prior to 1980, even though loan associations were kept under direct regulation. Hence, even though the liberalization from 1980 had been partly reversed, it is fair to say that deregulation of the bond market was well under way. The process would continue with a gradual decrease in placement requirements (both to liberalize the demand side and also because the requirement became redundant), and an easing of the supply side regulation. For instance, in 1983 the regulation of loan associations was relaxed further when the bounds for selling bonds to the non-financial private sector were removed.

Several important changes were introduced in 1985. Up until then, new bonds had a been required to have a minimum maturity of 12 months, but now *certificates* (bonds with a maturity up to 12 months) were allowed to be traded. Also, the placement requirement for banks was revoked from the 1. January 1985, and it was set to zero for life insurance companies and pension funds at the same time (and subsequently revoked in July 1985).

The very last regulations, which were bounds for the loan associations' lending for housing-purposes, primary industries and power plants, were removed from the 1st of July 1988.

2.3.3 Removal of exchange controls

The first step towards full removal of exchange controls was taken already in 1978. Before this banks had been given quantitative limits for their lending with institutions abroad. Now it was changed to a requirement that all authorized foreign exchange banks had to have an approximately zero total position (net spot and forward claims) at the end of every day. Hence lending abroad was unlimited for banks as long as their total position was close to zero. Initially this was only a trial system, but it was made permanent a few years later. This change had far-reaching consequences and has been claimed by some to have been one of the most important steps in the deregulation process (see Grønvik (1994, p. 207)). Banks could from now on borrow more extensively abroad and this weakened the traditional link between domestic deposits and domestic loans, reducing the authorities' ability to control the credit supply.

The liberalization of the exchange controls took a new step in June 1984. Domestic residents were from now on allowed to invest as much as they wanted in foreign stock markets, but

investments in non-listed stocks and in bonds were still regulated. Of other changes the licence requirement for direct investments in Norway was removed. But to reduce the supply of credit from abroad, foreigners' access to invest in bonds was withdrawn in November (earlier they could invest up to 1 mill. kroner).

In February 1985 domestic residents and companies were permitted to invest in foreign bonds denominated in foreign currency (but only up to a limit; 1 mill. kroner for private residents and up to 5 mill. for companies). Furthermore, in the autumn the same year the authorities removed the requirement of a license from domestic residents who took up a loan abroad (through the foreign exchange banks).

The liberalization of capital flows continued in 1989. In May foreigners were again allowed to purchase listed bonds in Norway and this time without any limits. In July the authorities gave domestic residents permission to buy shares in foreign securities funds. Finally, in December foreigners were allowed to issue bonds on the Norwegian bond market.

In 1990 a new set of foreign exchange regulations was presented, and this marked the end of the remaining foreign exchange control, even though the practical implications of this change were modest. Previously all transactions that were not permitted were forbidden. From now on the premise would be the opposite: all transactions were permitted unless they were forbidden.

2.3.4 Removal of quantitative regulations

Among the instruments for the credit policy listed in Section 2.1, the quantitative regulations were the last to be removed. The first easing of quantitative regulations was not the removal of one of the instruments in the list above but a step that was taken to make the banks more active in the mortgage market and thus reduce the importance of state banks. The banks agreed with the government to offer 6,000 mortgages with better conditions than normal mortgages, so-called PSV-loans.¹⁵ As a compensation the banks' lending bounds were increased by the same amount as all the PSV-loans in total.¹⁶

The next change was the *introduction* of new quantitative regulations, not removal of old. As already discussed in Section 2.2, an unregulated credit market based on direct loans between individuals had developed. Up until now, no regulations were designed to control the amount of credit in this market. In January 1983 this changed. From now on the guarantees that financial institutions issued for loans in the grey market were directly regulated. The guarantees could at the end of the year not have increased in real terms compared to the level in the 3rd quarter of 1982. With this the regulators were hoping to avoid that a tightening of ordinary credit only lead to a leak over to the grey market. This regulation was temporarily removed in the second half of 1984, but re-introduced in January 1986 limiting guarantees to the stay below the level

¹⁵PSV is an acronym for the Norwegian sentence 'På Spesielle Vilkår' which means 'On Special Terms'.

¹⁶This arrangement would continue for several years.

they ended at in 1985.

If we look at Figures 2.1-2.2 again, we see that both primary reserve requirements and additional reserve requirements were used frequently up until the mid 1980s. But from the beginning of 1984, the additional reserve requirements were removed, and this was actually intended to be a permanent removal. It was followed by a period of increased primary reserve requirements in one last attempt to only use indirect instruments to control the flow of credit. The job was not easy – it was a period of very strong growth in credit. The introduction of the regulation of guarantees issued for loans in the grey market (see description above) caused further difficulties. Since loans that were previously in the unregulated market were moved to the banks' balance sheets, it was hard for regulators to measure the real growth of credit. To dampen the growth, and despite previous intentions, additional reserve requirements were revived and given to both commercial and savings banks from early in 1986, and these were tightened again in the summer of 1986.

Apparently, the tightening of requirements did not have that much of an effect – see Figure 2.3. Since the requirements were not effective any longer, all primary reserve requirements were revoked in June 1987.¹⁷ The additional reserve requirements were removed the 9th of October 1987, marking the complete removal of quantitative regulation of banks. 1987 was also the last year the government "bothered" to put up a credit budget. The budget was abandoned from 1988, and one could argue that the budget had been mostly symbolic the last few years.¹⁸

When the year 1987 ended, there were very few regulations left. The guarantees issued by financial institutions for loans in the grey market were still regulated. That was also still the case for the lending from private finance companies and non-life insurance companies. However, these last regulations were not given much more time and both were removed from the 1st of July 1988. It can be argued that this marks the completion of the deregulation.

2.4 The development after deregulation

In the preceding description we have seen that Norway went through an extensive financial deregulation. As one tangible result of this, there was a boom in lending until the end of the mid 1980s, but reality caught up with Norwegian financial markets at the beginning of the 90s. We were about to witness a banking crisis, partly unleashed by the beginning of a decline in economic activity both in Norway and abroad, but also pushed forward by deeper, structural problems in an over-sized credit sector.

¹⁷Finance companies, who had been given a primary reserve requirement for their factoring loans in 1984, had to wait until October 1987.

¹⁸Instead of a credit budget the authorities started to announce a target zone for the desired level of credit growth. Every year a relatively wide zone for the planned growth in credit supplied to the private sector and municipalities would be spelled out, and the (now limited) credit policy instruments would be adjusted in an attempt to keep the growth within the bands.

2.4.1 The banking crisis

From around 1987/88 to 1992/93, Norway suffered a major banking crisis. Reinhart and Rogoff (2008) labels it as one of the world's "big five" banking crises in the post-war period. It forced many banks to close down and the government had to take over some of the largest banks in the country. This section will only give a brief sketch of the events. For more thorough presentations of the Norwegian banking crisis, see e.g. Norwegian Official Reports (1992) or Moe, Solheim, and Vale (eds.) (2004).

The first sign of weaknesses in the banking sector came in 1987 when the commercial banks (as seen in total) suffered net losses for the first time in many years. This was caused by both an increase in losses on loans but also a minor stock market collapse in October. The Oslo Stock Exchange index dropped by more than 40 %, pushing the index back to its levels of 1984/85. Gross losses for both commercial and savings banks had jumped from around 2,000 mill kroner in 1986 to just below 4,500 mill kroner in 1987. The tendency of increasing losses for the banks continued in 1988. 5 banks saw their entire capital base being wiped out that year and total losses for commercial and savings banks increased to 8,700 mill. kroner. The commercial banks were still facing a net loss and savings banks' profits were close to negative. The losses of finance companies were also increasing rapidly. The Commercial Banks' Guarantee Fund had to, for the first time in many years, guarantee for all the liabilities of a bank, the regional bank Sunnmørsbanken.

In 1989 the total losses for the banks amounted to 10,400 mill. kroner. It was especially the losses of savings banks that pushed the total up to its new level. However, the net results were actually better than in 1988 and they turned positive for both types of banks as gains from the stock market gave a boost to revenues, but there were large differences within the sector. The commercial bank Norion Bank became, at the 30th of October 1989, the first bank since 1923 to be put under administration. It was later decided to liquidate the bank. Several other banks also struggled. Many failed and their remaining parts were in most cases merged with larger banks. Finance companies had suffered big losses over the last years, but total losses in 1989 were smaller than in 1988 (1,400 vs. 2,000 mill. kroner). Loan associations were still in an acceptable situation, but their losses had also started rising.

The banking problems escalated in 1990 when the results of banks were the worst since WWII. This is also the year it is widely reckoned that the banking crisis erupted. Both commercial and savings banks had net losses, and their total gross losses ended at more than 12 billion kroner. For commercial banks this amounted to a net profit of -0.77 % of the average total assets. The sum of non-accrual loans was almost as large as the total losses and this gave warning about difficult times ahead. Several banks had grave problems and were either guaranteed by the Commercial Banks' Guarantee Fund or the Savings Banks' Guarantee Fund and many near-failing banks were merged with others. Finance companies continued to lose

money, but lost less than the year before. Loan associations also began to see larger losses, but continued to have a positive net result. The stock market peaked at a new record-high level in the beginning of August, only to drop again by almost a third by the end of the year. This made the financial situation for banks even worse. There was also evidence indicating that the amount of loans over the grey market had been reduced drastically.

Losses peaked in 1991. Banks faced total losses of almost 20 billion kroner. The Government Bank Guarantee Fund was established to provide loans to the two bank sectors' own Guarantee Funds such that they were able to prop up enough guarantees for all the banks that were in trouble. Later the fund was also allowed to invest directly in problem banks. To provide capital to relatively sound banks as well, The Government Bank Investment Fund was created to invest directly in banks on commercial terms. By 1992 it was clear that even though the losses had peaked in 1991, there were still problems remaining. The banking sector lost a total of roughly 12 billion kroner (i.e. still very large, but far less than in 1991) and the savings banks actually made a net profit during the year. The banks that had been supported by the government in the crisis had been given clear requirements with respect to cutting administrative costs and reaching a positive net result as soon as possible. 1992/93 can be regarded as the last years of the banking crisis. In 1993 the banks' net results had improved a lot since 1992, both due to lower losses and gains from increasing asset values. Economic activity had started to pick up in Norway, and was expected to do so internationally as well.

2.4.2 Changes in the capital requirement

After the quantitative regulations had been removed, the regulation that was left consisted mainly of the capital and the liquidity requirements (these are described in Section 2.1). Prior to the end of 1980s these requirements were not that important, as already noted. Primary reserve requirements normally made sure that liquidity requirements were met while capital requirements were not binding. Berg and Eitrheim (2009) argue that the regulators did not see it as necessary to enforce strict capital requirements in the 1970s and 80s. As a consequence, the capital requirement was relaxed on several occasions and in 1984 and 1987 changes were made to permit a larger share of a banks capital to consist of subordinated debt, which was in many cases raised internationally (again, see Berg and Eitrheim (2009)). However, from 1988 and onwards, the capital requirement was to become the main regulatory instrument.¹⁹

The first change came in 1988 when it was decided to let the savings banks face the same requirement as the commercial banks had already faced for a long time.²⁰ A more substantial

¹⁹The liquidity requirement was less important. It was also subject to some minor changes in 1988 and also during the 1990s, but in 2006 the formal liquidity requirement was replaced by a requirement for every institution to always have enough liquid assets to cover their liabilities when due.

²⁰The requirement was to keep the capital ratio at a minimum of 6.5 % of total liabilities (minus the capital itself and some near risk-free assets).

change occurred in 1991 when the Basel Accord was implemented, a set of regulations worked out by the Bank for International Settlements (BIS). This was a major step towards international coordination of bank regulation, but some variation of requirements across countries was permitted. The most important change relative to the old rules was the system of putting weights on different assets according to their presumed riskiness, in addition to a requirement of consolidation within groups of financial companies. Mortgages were given a favorable risk-weight of 50 % provided that the loan-to-value (LTV) ratio was less than 80 %. A bank's own capital would now have to be at least 8 % of this risk-weighted basis of calculation. The rules were in some ways stricter than the old, for instance due to a consolidation requirement, but the risk-weighted capital requirement itself was slacker than the old (again, confer Berg and Eitrheim (2009)). The new rules were to be implemented gradually over the course of a few years.

As a supplement to the capital requirements the Capital Adequacy Directive (CAD) was introduced in 1996.²¹ While the capital requirements that already existed were motivated by the desire to limit credit risk, CAD was meant to limit market risk. Financial instruments were given risk weights and then the weighted sum would form the basis of calculation for the extra capital requirement introduced (8 % of the basis). This requirement came in addition to the already existing capital requirement and capital used to cover the latter could not be used to cover the new one as well. The directive had a very modest effect. An update of the directive, CAD-II, came in 2000.

During the 1990s, the authorities judged that even though the capital requirements were fulfilled, core capital's share of the own capital was uncomfortably low, and they were also worried about the increasing share of mortgages extended with a very high LTV ratio.²² In 1998 two measures were taken to stop this. First, a house mortgage would only be given the 50 % risk weight if the LTV ratio was less than 60 %. Secondly, banks would only be allowed to take up new subordinated debt with fixed maturity if their core capital was at least 7 % of the basis of calculation for the capital requirement (this had already been informally practiced by Finanstilsynet). These two measures, together with a clear message to the banks about the need to tighten credit, dampened the credit growth in 1998. In 2001 the LTV limit was set back to 80 % (as it had been prior to 1998).

A final regulatory change came in 2007 with new capital requirements based on the Basel II Accord. This accord was much more complex and detailed than its predecessor. For all the details I refer to Basel Committee on Banking Supervision (2004). The Accord can be divided into three subcategories, or pillars. The first pillar is concerned with the minimum capital requirements. This was the equivalent to the old Basel Accord, but with some new and

²¹This directive was implemented in the whole European Economic Area.

²²The core capital is mainly the share capital, reserve funds, retained earnings and primary capital. That this ratio was low meant that a large part of the banks' capital consisted of subordinated debt, a less stable way of funding.

important features. Among these was the possibility for institutions to use their own risk models to calculate their capital requirement (so-called Internal Ratings Based). For those without their own model it was still a standard system with risk-weights assigned to different assets, much like the old system, but for mortgages with an LTV below 80 % the risk-weight was reduced from 50 % to 35 %. Furthermore, house mortgages with a greater LTV and other commercial loans (up to some maximum limit) were given a risk-weight of 75 %. The second pillar covered the rules regarding the supervisory review process. Lastly, the third pillar contains regulations to ensure market discipline through disclosure requirements. It is not realistic to give a good description of the Accord in this document, but for our purpose it's important to at least note that one important effect of the new rules was a reduced capital requirement for most financial institutions (Finanstilsynet, 2007b), mostly due to the changes within the first pillar.

3 The mortgage survey of Finanstilsynet

Finanstilsynet, the Norwegian Financial Supervisory Authority (FSA), has since 1999 conducted a yearly survey where the largest Norwegian banks report details regarding the first 100 home mortgages they extend after some date that year.²³ These details include the purpose of the mortgage, the loan-to-value ratio, time to maturity and whether the mortgage has a fixed or floating interest rate. The data for each survey is summarized in an annual report (Finanstilsynet, 1999b-2009b). In this chapter I will comment on the most important tendencies in the surveys.

3.1 Main tendencies

As this survey is conducted by the Norwegian FSA, the focus lies on monitoring the market for mortgages and look for instabilities that can lead to problems. The main development for the loan-to-value (LTV) ratios is given in Figure 3.1. There are no drastic shifts in the figure, but we see that there seems to have been a small tightening of LTV limits from 1997-2001, and after this a gradual easing up until 2006/07. In 2009 we observe a new and quite sharp tightening.²⁴ Note that these LTV ratios are based on not just mortgages that are to be used for purchasing a home but also mortgages used for refinancing or other purposes (using a house as collateral). One would probably expect the LTV limits for loans that are only for purchasing houses to be more skewed to the right. Starting from the surveys of 2007 and onwards, this is actually shown separately (with numbers starting in 2003) and we see that this is correct, but the *movements over time* are pretty similar to those of the "gross" LTV ratios.

Around 1998, Finanstilsynet started to worry that Norway was entering a new period of excessive credit growth and over-indebtedness. As noted in Chapter 2, one response from the authorities was to increase the risk-weight for loans with an LTV ratio between 60 and 80 percent to 100 % (instead of 50 %). This might have caused some of the shift we see in Figure 3.1. From 1997 to 1999, the survey shows that the share of loans with an LTV of 80 % or more decreased from 34.3 to 25.3 %. The share with 60-80 % stayed constant, making the category with less than 60 % increase by 9 percentage points to 39.1 %. In addition, the share of loans (based on the number, not value) that did not have to posit any form of extra security for their loan decreased from about 53 % to 43 % for loans from commercial banks with an LTV above 100% in 1998-2000. However, this share *increased* for savings bank loans in the same period (from 63 % to 70 %). By 2001, these shares shifted back to around their 1998-levels, at the

²³The exact date and time of the year the survey has been conducted has varied throughout the sample. The survey has actually been conducted since 1994, but the structure of the survey was quite different prior to 1999 (and there are no public reports of the surveys before this).

²⁴The numbers of 2008 do not reflect such a tightening as the survey conducted just around the dramatic events in the fall of 2008.

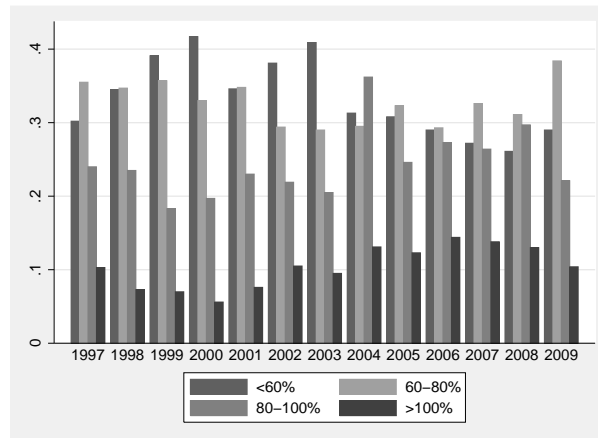


Figure 3.1: The development of loan-to-value (LTV) ratios. The share of loans with various LTV ratios (shares are based on the value of the loans). Source: Finanstilsynet (1999b-2009b).

same time as the LTV ratios started to shift upwards again.

Average time to maturity has shown an increasing trend from 1999 to 2008. From 1999-2003 it increased from 14.7 (14.3) to 17.6 (17) years for the mortgages from commercial (savings) banks. The average time grew for all levels of LTV ratios, and for the loans with an LTV above 100% it increased from 18.6 (16) to 21.8 (20.6) years, reducing the yearly costs of borrowing more than 100 % a fair amount. This general upward shift continued, and by 2006 the average time for loans with an LTV above 100% was around 23 years (on average for both types of banks), and in 2008 it was at 24 years. In 2009, the average time shifted a little bit back, but it was still above the 2006-level.

Starting with the survey from 2007, Finanstilsynet also reports the average loan-to-income (LTI) ratios, both on average and over the various LTV ratio groups. These numbers indicate a slightly increasing trend in average LTI ratios since 2006.

Home equity credit lines are a class of loans that were not covered by the mortgage survey before 2007. This is a relatively new innovation that has grown rapidly in usage and popularity since the mid 2000s. Data showing the extent of such credit lines are available from 2006, and they show that most of the growth in house mortgages has come from increase in credit lines.²⁵ Most of the credit lines have ceilings implying an LTV below 80 %. Finanstilsynet conducted a survey towards households that had taken up this type of loans, and the majority responded that the credit was to be used for redecoration/renovation of their home or for a new car or a cabin. This suggests that mortgage debt starts acting as a substitute for other kinds of debt.

²⁵This can also have contributed to the decrease in LTV ratios seen in Figure 3.1.

4 Model used to extract a CCI

This chapter explains the basic set-up for the model I will use to extract the credit conditions index. I follow the ideas of Fernandez-Corugedo and Muellbauer (2006) quite closely, but my exposition is in some respects more detailed. The CCI will be modeled as a common, unobservable variable in a system of equations for different credit indicators. We start this chapter by looking at the variables we will use as indicators.

4.1 Indicators for the credit conditions

We need variables that can serve as indicators for the credit conditions such that we rightfully can assume that the CCI has a significant impact on them. I do not have anything like the regional information and micro data in Fernandez-Corugedo and Muellbauer (2006), but I have the same two *aggregate* variables: households' stock of secured and unsecured debt. My series are a bit crude and have been constructed on the basis of total loans to households and housing loans to households, see Appendix A for the details. Both variables are available for the period 1975Q4 - 2009Q1. As we saw in Chapter 1, the results from Blake and Muellbauer (2009) indicate that it might be possible to estimate a meaningful CCI with only two equations.

The series for secured and unsecured debt are illustrated in Figure 4.1. Both variables have a small peak around 1980, and then grow rapidly until around 1990 and the onset of the banking crisis. Starting in the middle of the 1990s, secured debt enjoys a very strong growth rate, while the level of unsecured debt stagnates somewhat from 2005.

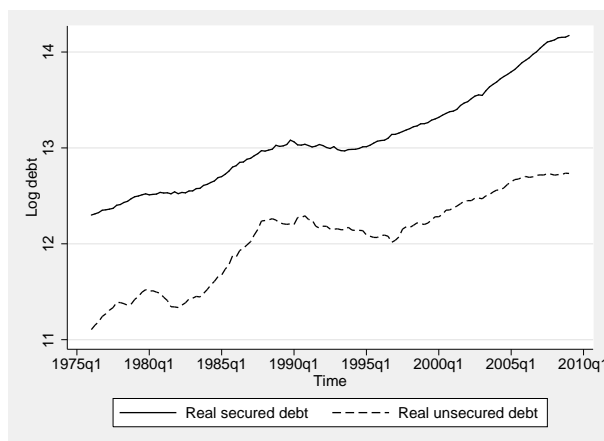


Figure 4.1: Real secured and unsecured debt, 1975Q4-2009Q1. Log scale.

4.1.1 Expected effects of the CCI

The main premise for the model I will formulate is that changes in the credit conditions have an important effect on the level of secured and unsecured debt. Based on the information in

Chapters 2-3, when are we expecting the credit conditions to matter?

In the late 1970s, the most dominant policy change was the contractionary credit policy of 1978-79, making me believe that credit conditions should have tightened and thus reduced the level of credit available *ceteris paribus*. It is not obvious how strong effect we should expect from the reduction in exchange controls in 1978, but it runs counter to the negative impact from the contractionary policy and could have partly neutralized it.

If there is a negative shift in the late 70s, some of it might still linger on in the early 1980s, but by the time of 1983/84 I expect a sharp positive shift in the credit conditions, contributing to the strong growth in credit up until the end of the 80s. It is in this period the bulk of deregulation took place and most regulations were removed by 1987. During the banking crisis I do not believe there are any institutional changes that are able to affect the picture significantly. The crisis will most likely lead to a tightening in the credit conditions, as banks become more careful both during the crisis and also in the aftermath. Hence, from around 1988/89, credit conditions should tighten.

After the crisis, say, from around 1995, I expect to see no significant tightening of credit conditions, maybe except for a shift in 1998 (due to the temporary increase in risk weight for loans with an LTV between 60 and 80 %) and in late 2008/early 2009 (due to the financial crisis). As documented by Finanstilsynet (1999b-2009b), it seems to have been a trend towards easing of credit conditions, manifested in an upward trend in the LTV ratios, increased average time to maturity and the introduction of home equity credit lines. The introduction of the Basel I and II accords (in 1991 and 2007) might also have contributed to an upward trend, as well as the continued reduction in exchange controls and internationalization of capital flows. This leads me to expect a gradual easing of credit conditions during the last part of our sample (up to the financial crisis).

There exist some other studies which look at the process of financial deregulation. Kamin-sky and Schmukler (2003) contains a wide survey of how financial liberalization has taken place in 28 different countries, including Norway. The authors distinguish between the liberalization of the capital account, the domestic financial system and stock markets. Periods are labeled as either repressed, partially liberalized or fully liberalized regimes. In their composite index (taking all three measures into account) the Norwegian financial market was partially liberalized from September 1985, and fully liberalized from January 1988. This fits well with the information presented here.

4.1.2 Other explanatory variables

What other variables can be expected to affect secured and unsecured debt? Fernandez-Corugedo and Muellbauer (2006) contains a careful discussion of this and some of their suggestions are:

- A demographic variable measuring the proportion of potential first-time buyers

- Income
- Change in the unemployment rate
- Wealth, divided into liquid financial, illiquid financial and housing wealth
- Nominal interest rate
- Real interest rate
- Expected income and expected interest rates

I choose to follow the main arguments of Fernandez-Corugedo and Muellbauer (2006) with respect to which variables to include but with some modifications.

First, and similar to the procedure of Fernandez-Corugedo and Muellbauer (2006), I "deflate" both the debt variables and all the wealth variables by the level of income. This is done to extract trends in the variables that are due to growth in the economy. Hence, I will end up formulating a model explaining the movements in secured and unsecured debt *relative to income*. When a variable X has been deflated by income I denote that variable by \hat{X} . The effect of deflating the debt variables is illustrated in Figure 4.2. For secured debt we see clear effects through the entire sample, while the effects for unsecured debt are visible after 1990. We denote secured and unsecured debt as SD and USD , respectively, making the notation for the deflated series \hat{SD} and \hat{USD} .

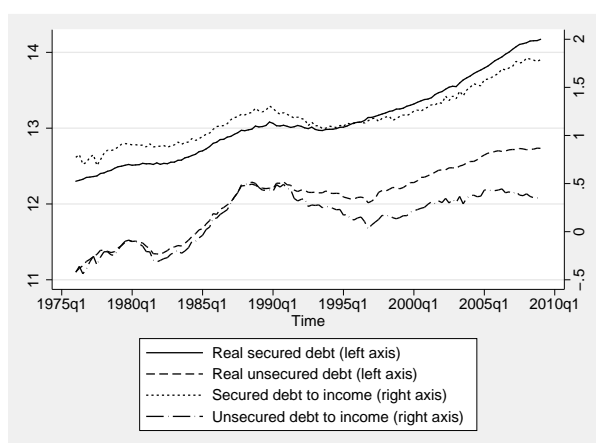


Figure 4.2: Debt variables deflated by income, 1975Q4-2009Q1. Log scale.

As a demographic variable I will use the proportion of individuals in the age group 20-39, relative to all persons aged 20-74 (call this variable FTB). To control for shifts in income I include income "per capita" ($INCCAP$). In the income variable I have subtracted dividends. These are left out as it only adds noise to our system due to a tax-change that caused a surge in dividends paid out in the years prior to 2006, followed by dramatic drop. To obtain per capita

numbers I divide by the number of persons of the age of 20-74. I use the same income variable (without dividing by the population) to deflate debt and wealth variables.

Wealth is included as liquid wealth (*LIQ*, defined as notes, coins and deposits), moderately liquid wealth (*MLIQ*, defined as bonds, stocks, loans and other claims) and housing wealth (*HW*).²⁶ The real interest rate net of taxes is also included (r). The change in the nominal interest rate (Δi) is added to capture the cash-flow effect for households from changes in the nominal interest rate level. Changes in the unemployment rate (Δu) are included to capture uncertainty. Expected growth in interest rates (*iexp*) is included as the difference between the rate on Norwegian 10-year government bonds and the money market rate. Finally, I have a variable capturing expected income growth (*incexp*). This is an estimate of the log deviation between the permanent income and current income of households. It is based on an assumption that households estimate their future income using a simple OLS model. Appendix A documents all the data series.

The discussion in Fernandez-Corugedo and Muellbauer (2006) also includes arguments about how they expect all these variables to affect secured and unsecured debt. Some but not all of the arguments below coincides with theirs.

I expect *FTB* to have a positive effect on both variables. The age-group 20-39 is, broadly speaking, the group of first-time buyers. When it increases, there is a greater need for mortgages. A negative impact might arise from the fact that first-time buyers probably take up smaller mortgages than agents closer to their prime age, but as our age group is so broad this should not be a large effect. Furthermore, this age group might use unsecured debt to smooth income in anticipation of higher wages in the future. The partial effect of higher income per capita is expected to be positive for secured debt because higher income allows you to buy a greater house (given some degree of credit constraints to begin with). On the other hand, one would expect income per capita to have a negative impact on unsecured debt since that reduces the need for taking up expensive loans. At the same time, higher income permits you to service a larger amount of debt, and if you really are credit constrained, you might choose to increase your stock of expensive debt if this is your only alternative. Hence the income effect on unsecured debt is ambiguous. The change in the unemployment rate is included as a proxy for income uncertainty and is expected to have a negative impact on secured debt to income. It should also affect unsecured debt negatively but the net effect is ambiguous since unemployed might use unsecured debt to smooth their income. Parts of the effect is already captured by the income term, but maybe not all of it.

The wealth to income variables are expected to affect the debt to income ratios through one main mechanism: For secured debt, the wealth variables can be used as collateral. They

²⁶ Illiquid wealth (defined as insurance technical reserves) is left out, mainly due to its somewhat diffuse economic interpretation.

can also, to a varying degree, serve as "back-up" for unsecured debt; some households will feel more comfortable taking up unsecured debt when they know they have assets that can be sold if necessary. In addition, the wealth variables might affect the need for debt. When house prices go up, this increases the need for debt, making the positive effect from housing wealth stronger. More liquid wealth might have a partially negative effect on debt, and especially unsecured debt, since that reduces the need for taking up expensive loans. But I do not believe that this effect will dominate the "back-up" effect. Hence I expect in general all the wealth variables to have a positive effect on both secured and unsecured debt to income.

An increase in the nominal interest rate is assumed to affect secured debt negatively. This effect kicks in for unsecured debt as well, but here it might also be a positive substitution effect as it becomes harder to get secured debt when the interest rate is high. This makes the sign of Δi ambiguous for unsecured debt. A higher real interest rate should have a negative effect for both types of debt. It makes loans more costly and motivates inter-temporal substitution. There is an off-setting effect from the fact that the return on the wealth of households increase in value, but this effect should be captured by the wealth terms (if their value increases) and the income variable (since net interest income is a part of it). Higher expected growth in interest rates should decrease both types of debt. Higher expected income should have a positive effect on both types of debt.

4.2 The model

I first formulate a vector auto-regressive (VAR) model that includes all the variables in my dataset. VARs are, as shown below, very general formulations that in principle treats all the variables in the system as endogenous. I will guide us through the assumptions that are needed to reduce this system to one with two semi-reduced form equations for secured and unsecured debt to income. Even though I only impose restrictions without testing for them, I think it is useful to begin with the general formulation in any case, such that the assumptions underlying the model are made as clear as possible. Most tests must be skipped since the CCI itself is estimated from the data, making it very difficult, if not impossible, to perform a full statistical analysis. This also makes it necessary to simplify the short-run dynamics of the model to avoid an extremely large number of parameters.

As shown in Appendix B, several of the variables we will use in the analysis can be interpreted as $I(1)$ variables – that is, they are non-stationary variables of degree 1. This forces us to take into account some considerations that are not relevant in the stationary world. In general, regressing an $I(1)$ variable on other $I(1)$ variables might lead to a phenomena called *spurious regressions* (see Granger and Newbold (1974)). The point is that even if two $I(1)$ variables are completely *uncorrelated*, the coefficient estimate from a regression of one of them on the other

nevertheless converges asymptotically towards a non-zero value. You might end up detecting a relationship even though there is no such present (hence the term spurious regression). However, not all regressions that involve non-stationary variables are spurious. This is where the concept of cointegration enters. A set of $I(1)$ variables are said to be *cointegrated* if a linear combination of these is a stationary variable (see e.g. Hamilton (1994, Chapter 19)). If, for instance, aggregate consumption is always a given proportion of income (times some multiplicative noise), then the difference between the log of the two series is always just a constant plus some noise. Even though both (log of) consumption and income are found to be non-stationary, the difference between the two will be stationary. Such a linear combination of the variables is usually referred to as the cointegrated relationship. Furthermore, a set of q variables can have as many as $q - 1$ cointegrating vectors, such that finding cointegration between more than 2 variables raises the issue of finding the number of such vectors as well.

There exist well documented methods for testing the number of cointegrating vectors. For a maximum likelihood method see Hamilton (1994, Chapter 20) which describes the Johansen method due to Johansen (1988). Within the same framework there also exist tests for which variables in the relationship that are exogenous with respect to the parameters of the cointegrating vector. We will see below how these concepts are related to our model.

4.2.1 Defining equations

Let us define three vectors of variables:

$$y_t = \begin{pmatrix} \hat{s}d_t \\ \hat{u}s\hat{d}_t \end{pmatrix}, \quad x_t = \begin{pmatrix} inccap_t \\ \hat{l}i\hat{q}_t \\ m\hat{l}i\hat{q}_t \\ \hat{h}\hat{w}_t \\ r_t \\ FTB_t \end{pmatrix}, \quad v_t = \begin{pmatrix} \Delta i_t \\ \Delta u_t \\ incexp_t \\ iexp_t \end{pmatrix}$$

where lowercase letters imply that it is the natural logarithm of the variable (except for i , r , u , $incexp$ and $iexp$ which are in rates). $inccap_t$ has been deflated with the CPI. Let k_x denote the row dimension of x and k_v be that of v . We have observations of y_t , x_t and v_t for $t = 0, 1, \dots, T$, where 0 refers to the fourth quarter of 1975 and $T = 133$ is the first quarter of 2009. Let the credit conditions index at time t be given as CCI_t . For now I will treat CCI_t as any other variable. In Section 4.2.2 we will consider its functional form.

I assume that y , x and CCI follow a VAR process of lag-order p where v enters contemporaneously as an exogenous variable. This means that the variables in v are assumed to only affect the short-run dynamics, and such a simplification makes sense: we assume that the effects from changes in the unemployment rate, changes in the interest rate, expected income

growth and expected growth in interest rates are all "neutral" in the long run. The VAR is given by (ignoring constant terms for now):

$$\begin{pmatrix} y_t \\ x_t \\ CCI_t \end{pmatrix} = A'_0 v_t + \sum_{i=1}^p A'_i \begin{pmatrix} y_{t-i} \\ x_{t-i} \\ CCI_{t-i} \end{pmatrix} + e_t^* \quad (4.1)$$

for $t = p, p+1, \dots, T$ where A_0 is a $k_v \times (3+k_x)$ coefficient matrix and A_i is a $3+k_x$ square matrix for $i = 1, \dots, p$. The column vector e_t contains two bi-normally distributed errors. I assume that the vector of disturbances e_t^* is independently and identically normally distributed according to:

$$e_t^* \sim N(0, \Omega^*) \quad (4.2)$$

Ω^* denotes the $(3+k_x) \times (3+k_x)$ variance-covariance matrix. We note that, as was intended, this formulation permits in principle all the variables in y_t , x_t and CCI_t to be endogenous with respect to the system.

It is useful to rewrite (4.1) in the following way:

$$\Delta \begin{pmatrix} y_t \\ x_t \\ CCI_t \end{pmatrix} = A'_0 v_t + \Pi' \begin{pmatrix} y_{t-1} \\ x_{t-1} \\ CCI_{t-1} \end{pmatrix} + \sum_{i=1}^{p-1} \Pi'_i \Delta \begin{pmatrix} y_{t-i} \\ x_{t-i} \\ CCI_{t-i} \end{pmatrix} + e_t^* \quad (4.3)$$

with the new coefficient matrices defined as:

$$\begin{aligned} \Pi' &= \sum_{i=1}^p A'_i - I_{3+k_x} \\ \Pi'_i &= - \sum_{j=i+1}^p A'_j \end{aligned}$$

As shown in Appendix B, the variables of y_t as well as $inccap_t$, \hat{liq}_t , \hat{mliq}_t , \hat{hw}_t and FTB_t can all be interpreted as I(1) variables. Let us also assume the CCI_t can be regarded as I(1). The remaining variables can be interpreted as I(0). Since I(1) variables become I(0) when differenced, $\Delta \begin{pmatrix} y_{t-1} & x_{t-1} & CCI_{t-1} \end{pmatrix}'$ is a vector of I(0) variables, ignoring the small inconsistency of letting r enter as well.²⁷ Hence the only term consisting of I(1) variables in (4.3) is $\Pi' \begin{pmatrix} y_{t-1} & x_{t-1} & CCI_{t-1} \end{pmatrix}'$. If the model is to be able to explain its left-hand side (LHS) variables, it is necessary that the equations in the system are *balanced* (Granger, 1990, p. 12-13),

²⁷ r is permitted to be a part of the cointegration space. In principle, adding an I(0) variable (such as r) to a combination of I(1) variables will not help you achieve cointegration, but as long as r has some non-stationary "tendencies" in parts of the sample this might be enough to get cointegration. In addition, there will in many applications be economically relevant to include I(0) variables in levels form since we want to interpret the cointegrating relationships as steady-state theoretical relationships which can include variables of both types.

meaning that if the LHS is $I(0)$, the right-hand side (RHS) must be $I(0)$ as well. This implies that the system in (4.3) makes logical sense only if $\Pi' \begin{pmatrix} y_{t-1} & x_{t-1} & CCI_{t-1} \end{pmatrix}'$ is a vector of $I(0)$ variables.

This is where the concept of cointegration enters. We remember that q different $I(1)$ variables are cointegrated if there exist at least one linear combination of them that is stationary. Furthermore, these q variables can be involved in at most $q - 1$ different cointegrated relationships. It is clear that the only way $\Pi' \begin{pmatrix} y_{t-1} & x_{t-1} & CCI_{t-1} \end{pmatrix}'$ can consist of stationary terms, besides the zero vector possibility, is if it consists solely of cointegrated relationships. Basically, this is the essence of *Granger's Representation Theorem* (Engle and Granger, 1987). It says that if there exists cointegration between a set of variables, then a vector error-correction model (VECM) of the type in (4.3) can be formulated, and vice versa. Hence, given cointegration, the formulation in (4.3) is valid, and if (4.3) is the true process, then there exist cointegration (as long as Π is not a zero matrix).

Now even if the "presence" of cointegration is settled, we need to find out how many cointegrated relationships there are. This question is closely related to the *rank* of Π . The rank of Π is the number of linearly independent rows (or columns) in Π , and it is denoted $r(\Pi)$. This should tell us that the number of cointegrated relationships in the system (4.3) equals $r(\Pi)$.

The intuition is quite straight forward. If $r(\Pi) = 0$ then Π must be the zero matrix. The system is balanced since no $I(1)$ variables enter any longer, and there exist no cointegration since no linear combinations of y , x and CCI affect their respective growth rates. If on the other hand $r(\Pi) = c$ with $0 < c < 3 + k_x$, then there are c different cointegrated relationships. This means that c different linearly independent columns of Π will, in product with $\begin{pmatrix} y_{t-1} & x_{t-1} & CCI_{t-1} \end{pmatrix}'$ result in $I(0)$ variables. The remaining terms of $\Pi' \begin{pmatrix} y_{t-1} & x_{t-1} & CCI_{t-1} \end{pmatrix}'$ will be linear combinations of these products.²⁸

If we were to perform a state of the art Johansen analysis, it would at this stage have been the time to choose the lag length p such that we got a well-specified model (judged by the properties of the residuals) and then use some tests in order to determine the rank of Π (the number of cointegrated relationships). But, as already noted, this is not possible to do since CCI is itself being estimated. Instead I choose p to simplify the short-run dynamics and I choose $r(\Pi)$ to get a relatively sensible model. My assumption is that the variables in y , x and the CCI form two cointegrated relationships, which subsequent restrictions will make sure are the long-run equations for secured and unsecured debt to income. Let us impose the restrictions:

Restriction #1: Set $p = 1$ in order to simplify the short-run dynamics.

Restriction #2: Set $r(\Pi) = 2$ to get two cointegrated relationships.

²⁸Clearly, $r(\Pi) \neq 3 + k_x$ since q variables can only be linearly combined in $q - 1$ unique ways as long as the combination must involve at least two variables. $r(\Pi) = 3 + k_x$ is only possible if all the variables in y , x and the CCI are $I(0)$.

The next step is to use that the matrix Π' can be written as the product $\alpha\beta'$ where α and β are both $3 + k_x \times r(\Pi)$ matrices (see the argument in e.g. Hamilton (1994, Chapter 19)). With $r(\Pi) = 2$ we have:

$$\Pi' = \alpha\beta' = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{x1} & \alpha_{x2} \\ \alpha_{c1} & \alpha_{c2} \end{pmatrix} \begin{pmatrix} \beta_{11} & \beta_{12} & \beta_{1x} & \beta_{1c} \\ \beta_{21} & \beta_{22} & \beta_{2x} & \beta_{2c} \end{pmatrix}$$

where α_{xi} and β'_{xi} are column vectors of order k_x for $i = 1, 2$. It is very useful to write Π' as such a matrix product since it illuminates interesting aspects of the structure of Π' . β is usually referred to as the *cointegrating vectors*, defining the two linear combinations that result in cointegration. α is often called the *loading matrix*. It can be interpreted as the matrix with the coefficients attached to the cointegrated relationships in the various equations. Interpreting (4.3) as a VECM implies that α contains the error-correction coefficients.

Expressing Π' in this way allows us to be very precise about what restrictions we impose. First we note that the long-run system is not yet identified. The reason is that even though we only have two equations left, these contain the exact same variables on the RHS. To secure identification we need at least one unique variable in each equation and we also need to "set the scale" in each equation. As exclusion restrictions I assume that the variables in y are excluded from one equation each. Below we will see that the two equations are interpretable as long-run equations for secured and unsecured debt. Hence, this restriction implies that the long-run level of secured (unsecured) debt does not depend on unsecured (secured) debt. This may be a too strong assumption – see the discussion in Section 6.1.4. Identification is secured as soon as we choose a normalization in the two columns of β in order to set the scale. This is an innocent action as long as the true value of the coefficient we normalize with respect to is different from zero. The restrictions we impose are:

Restriction #3: Set $\beta_{12} = 0$ to exclude *usd* from the first relationship.

Restriction #4: Set $\beta_{21} = 0$ to exclude *sd* from the second relationship.

Restriction #5: Set $\beta_{11} = -1$ to set the scale in the first relationship.

Restriction #6: Set $\beta_{22} = -1$ to set the scale in the second relationship.

Even if it had been possible to observe *CCI*, it would still not be possible to test the validity of these 4 restrictions. This is the nature of identifying restrictions; you need them in order to identify the long-run system.

Our next step is to simplify the system further. As we see, the variables in y , x and the *CCI* are all still potentially endogenous with respect to the system: they still depend on the

cointegrated relationships. A variable is exogenous to the system only if we can assume that its α -coefficients are zero. Imposing exogeneity is something one would have to test for in normal circumstances, but here I will just assume that the variables in x and the CCI are exogenous to the system. In addition I will impose the restrictions that each debt variable is exogenous with respect to the other debt variable. The restrictions are:

Restriction #7: Set $\alpha_{xi} = 0_{kx}$ for $i = 1, 2$ to get x as exogenous to the system.

Restriction #8: Set $\alpha_{ci} = 0$ for $i = 1, 2$ to get CCI as exogenous to the system.

Restriction #9: Set $\alpha_{12} = 0$ to get usd as exogenous to the first relationship.

Restriction #10: Set $\alpha_{21} = 0$ to get sd as exogenous to the second relationship.

where 0_{kx} is the column vector of order k_x containing only zeros.

Imposing restrictions #1-10 on the system in (4.3) gives the final equations that we are going to work with. Based on the exogeneity assumptions it is clear that the two cointegrated relationships in principle represents *long-run equations* for the two endogenous variables that are left (actually they represent these variables' deviation from long-run equilibrium – see below). These restrictions also legitimate that we only focus on a subset of the complete system, namely the equations for these two endogenous variables. This can be presented as:

$$\Delta y_t = A'_R v_t - \alpha_R (y_{t-1} - \beta'_{Rx} x_{t-1} - \beta'_{Rc} CCI_{t-1}) + e_t \quad (4.4)$$

where A_R is the matrix containing the two leftmost columns of A_0 and α_R is the matrix of the two uppermost rows of α . e_t is the subset of the two uppermost elements of e_t^* and it will be binormally distributed $N(0, \Omega)$ for some 2×2 variance-covariance matrix Ω .²⁹ We have the long-run coefficients of x and CCI defined as:

$$\beta'_{Rx} = \begin{pmatrix} -\beta_{x1}/\beta_{11} \\ -\beta_{x2}/\beta_{22} \end{pmatrix} \quad (4.5)$$

$$\beta'_{Rc} = \begin{pmatrix} -\beta_{c1}/\beta_{11} \\ -\beta_{c2}/\beta_{11} \end{pmatrix} \quad (4.6)$$

As already emphasized, the two elements of $(y_{t-1} - \beta'_{Rx} x_{t-1} - \beta'_{Rc} CCI_{t-1})$ are the two cointegrated relationships. These can be interpreted as *the endogenous variables' deviation from long-run equilibrium*, making $\beta'_{Rx} x_{t-1} + \beta'_{Rc} CCI_{t-1}$ the two long-run equilibria. We see that the deviations will affect the growth rates of the endogenous variables, making the variables error-correct as long as the α -terms are positive. This is why such models are called error-correction

²⁹The form of Ω will naturally depend on Ω^* , but that is not of interest to specify further in our setting.

models. Finally, if we are to estimate the system in (4.4), we need not worry about the problem of spurious regressions mentioned earlier as long as our assumption of cointegration holds, since this leaves only $I(0)$ elements left in our model.

4.2.2 Defining the CCI

Until now we have treated the CCI as any other variable, despite that it is a variable it is not possible to observe. In order to estimate the CCI we need to specify a functional form for it. Assume that CCI can be represented as a piecewise linear spline function plus a function of other variables:

$$CCI_t = \sum_{i=1}^T \delta_i qd_{it} + h(cr_t) \quad (4.7)$$

for $t = 1, 2, \dots, T$ where qd_{it} is a dummy for quarter i which takes the value 0.25 for $t \geq i$ and zero otherwise. The spline-function is completed by assuming $\delta_i = \delta_j$ for all pairs (i, j) belonging to the same year (i.e. the spline function has constant slope within each year). cr_t represents other variables that we allow to affect the credit conditions. To be clear, this formulation implies that the CCI in different periods is defined as:

$$\begin{aligned} CCI_{t_0} &= 0.25\delta^0 + h(cr_{t_0}) \\ CCI_{t_1} &= 0.50\delta^0 + h(cr_{t_1}) \\ &\vdots \\ CCI_{t_5} &= \delta^0 + 0.5\delta^1 + h(cr_{t_5}) \end{aligned}$$

where it is assumed that t_0 refers to the first quarter of year 0 and δ^j is the coefficient of the dummy variables belonging to year j .

In the function h I want to add other variables that we are assuming only affects the credit indicators through their impact on CCI. One such variable is the primary reserve requirement for commercial banks in the south.³⁰ Let the primary reserve requirement in period t be given as $primres_t$. This will be the quarterly average based on the monthly observations (as presented in Figure 2.1). I also add a variable that signals whether additional reserve requirements were active or not. The variable takes the value 1 if additional reserve requirements were active through the entire quarter, $2/3$ if they were active in two thirds of the quarter and $1/3$ if they were active in only one month. Let this variable at time t be given as $addreq_t$ (see Figure 2.2 to see when the requirements were active).

³⁰We choose the reserve requirement for the commercial banks instead of that for savings banks because it was changed more frequently, and can therefore potentially serve as a variable signalling the general policy of the government as well. In addition, the patterns of commercial and savings banks' requirements are quite similar, so it does not matter that much which requirement we choose.

Define now $cr_t = (\text{primres}_t, \text{addreq}_t)'$. I assume that the requirements affect CCI linearly such that the h -function can be given as:

$$h(cr_t) = \xi' cr_t \quad (4.8)$$

where ξ is a 2×1 matrix. Gather all the δ_i 's in a column vector of order T (with the within-year restriction imposed) δ and all the quarter shift-dummies in a column vector qd_t . Let $(\delta, \xi)' = \eta$ and $(qd_t, cr_t)' = c_t$ such that the final definition of the CCI becomes:

$$CCI_t = \eta' c_t \quad (4.9)$$

4.2.3 Normalization

Use the definition of CCI_t from (4.9) to substitute for CCI_t in (4.4). Since the CCI is a function of coefficients that we estimate, it is clear that we cannot estimate both elements of the vector β_{Rc} . However, since the CCI is without scale there are no problems involved in normalizing the value of this index somehow. I choose units for CCI such that the long-run effect of a unit increase in CCI results in a unit increase in the long-run value of sd (given the level of income), i.e. the semi-elasticity is equal to one (since a 1/100th increase in CCI leads to a one percentage increase in SD). This means that we should interpret an increase in the CCI as an *easing of credit conditions*. The identified version of (4.4) then becomes:

$$\Delta y_t = A'_R v_t - \alpha_R \left(y_{t-1} - \beta'_{Rx} x_{t-1} - \begin{pmatrix} 1 \\ \gamma \end{pmatrix} \eta' c_{t-1} \right) + e_t \quad (4.10)$$

where

$$\gamma = \beta_{c2}/\beta_{c1}$$

γ can be interpreted as measuring the impact on usd of a change in CCI that led to a unit increase in sd (again for a constant level of income).

4.2.4 Estimation

Once the data for y_t , x_t and c_{t-1} are available, we are ready to estimate the model. We will estimate:

$$\Delta y_t = A'_R v_t + B'_0 y_{t-1} + B'_1 x_{t-1} + \begin{pmatrix} 1 \\ \tilde{\gamma} \end{pmatrix} B'_2 c_{t-1} + e_t \quad (4.11)$$

for $t = 1, \dots, T$ where the long-run coefficients in (4.10) are identified through the equations:

$$B'_0 = -\alpha_R \quad (4.12)$$

$$B'_1 = \alpha_R \beta'_{Rx} \quad (4.13)$$

$$B'_2 = \alpha_{11} \eta' \quad (4.14)$$

$$\tilde{\gamma} = \frac{\alpha_{22}}{\alpha_{11}} \gamma \quad (4.15)$$

Since the errors are correlated and the RHS variables differ between the two equations, this looks like a Seemingly Unrelated Regressions model (SUR model), as originally developed by Zellner (1962). Inspecting the model further we realize that the model is nonlinear in some of the parameters such that what we have is a nonlinear SUR model.

When ordinary SUR models are estimated it is common to apply either Feasible Generalized Least Squares (FGLS) or Maximum Likelihood (ML) estimation. Most statistical softwares provide standard codes for estimating SUR models using FGLS, while an explicit ML code for SUR models using the statistical software Stata is provided in Gould, Pitblado, and Sribney (2003). FGLS codes for SUR models are usually, at least in Stata (the command `sureg`), based on a two-step procedure as described in Greene (2003, p. 340-47). This procedure has a clear connection to ML since iterating the steps will produce ML estimates. However, this is not the path taken in "pure" ML models as in Gould, Pitblado, and Sribney (2003). In these models a more direct formulation is used to attack the problem.

When estimating a nonlinear SUR model, we face basically the same alternatives. We can either apply Feasible Generalized Nonlinear Least Squares (FGNLS) or direct ML estimation. In Stata the command `nlsur` applies FGNLS to estimate such models and it also has an option that sets it to iterate (making it produce ML estimates). I have not found any examples of pre-written ML codes for nonlinear SUR models, but it is in principle possible to rewrite the codes from Gould, Pitblado, and Sribney (2003).

In this thesis I have chosen to apply pure ML estimation. I will derive the entire ML framework necessary in order to estimate (4.11) and then modify the codes from Gould, Pitblado, and Sribney (2003) (and also write some codes on my own) in order to get a Stata command tailor-made for our purposes. It must be admitted that the entire ML code was finished before I realized that the command `nlsur` existed (it was introduced with the release of Stata version 10 in 2007). Still, developing the ML framework is a useful exercise and I am hoping that estimating the model in a pure ML framework might make it more flexible for some types of extensions in the future. There might also be some problems with the `nlsur` command that I am not aware of which will strengthen the argument for our own ML code further.³¹

³¹As one small "victory" we should also note that `nlsur` seems to have one, albeit small, problem that the command I developed does not have. It seems to be sensitive to the choice of normalization (as that made in

5 Estimation method

This chapter derives the framework that is needed to estimate the model in (4.11) using maximum likelihood (ML) estimation. This involves finding the likelihood functions we are going to need and also some of their properties. What we really seek to maximize is the likelihood function derived in Section 5.1. However, it turns out that we need to make a de-tour which involves maximizing both a *concentrated* likelihood function and a *gamma-concentrated* likelihood function. These functions are derived in Section 5.2 and 5.3. Finally, Section 5.4 explains how we implement the model in a routine which we can use in Stata to estimate the whole system.

5.1 Likelihood function

Before we start, let us rewrite the system (4.11) in a more compact fashion. This is useful when we derive the succeeding results, and it is also in line with the Stata syntax. We define:

$$\begin{aligned}\Phi'_1 &= (A'_{R1}, B'_{01}, B'_{11}) \\ \Phi'_2 &= (A'_{R2}, B'_{02}, B'_{12}) \\ z_t &= (v_t, y_{t-1}, x_{t-1})'\end{aligned}$$

where A_{Rj} is the j th column of the matrix A_R and B_{ij} is the j th column of the matrix B_i ($i = 0, 1$ and $j = 1, 2$). (4.11) is then:

$$\Delta y_t = \begin{pmatrix} \Phi'_1 z_t + B'_2 c_{t-1} \\ \Phi'_2 z_t + \tilde{\gamma} B'_2 c_{t-1} \end{pmatrix} + e_t \quad (5.1)$$

The basis for estimating the model in (5.1) is to choose estimates for our parameters Φ_1 , Φ_2 , B_2 , $\tilde{\gamma}$ and Ω such that we maximize the *likelihood of observing the given outcome we have observed*.

I have T observations of $\{\Delta y_t, z_t, c_{t-1}\}$. The only probabilistic assumption I have made concerns the distribution of the error terms e_t . I have assumed that each vector e_t is identically, independently distributed according to $N(0, \Omega)$. This implies that the probability density function (pdf) of e_t is given as:

$$f_e(e_t) = (2\pi)^{-1} |\Omega|^{-1/2} \exp \left[-\frac{1}{2} e'_t \Omega^{-1} e_t \right] \quad (5.2)$$

Section 4.2.3) such that changing the normalization *does* have an effect on the estimates. This is only a problem when I add the command that asks the code to iterate (to produce ML estimates instead of FGNLS). See Section 5.4.2 for how the problem was solved in the ML code developed here.

If we now condition Δy_t on z_t and c_{t-1} , the only random element in Δy_t is e_t . Hence, conditional on z_t and c_{t-1} the pdf of $\Delta y_t - E(\Delta y_t)$ is given as:

$$f_y(\Delta y_t - E(\Delta y_t); z_t, c_{t-1}) = (2\pi)^{-1} |\Omega|^{-1/2} \exp \left[-\frac{1}{2} r_t' \Omega^{-1} r_t \right] \quad (5.3)$$

where

$$r_t = \begin{pmatrix} \Delta y_{1t} - \Phi_1' z_t - B_2' c_{t-1} \\ \Delta y_{2t} - \Phi_2' z_t - \tilde{\gamma} B_2' c_{t-1} \end{pmatrix} \quad (5.4)$$

Since the errors are uncorrelated and since $Pr(\Delta y_t = z) = Pr(\Delta y_t - z = 0)$, we find the probability of observing our given outcome $\{\Delta y_t\}_{t=1}^T$ as:

$$g_Y(\{\Delta y_t\}_{t=1}^T; \{z_t, c_{t-1}\}_{t=1}^T) = \prod_{t=1}^T \frac{1}{2\pi} |\Omega|^{-1/2} \exp \left[-\frac{1}{2} r_t' \Omega^{-1} r_t \right] \quad (5.5)$$

When applying ML this equation is turned around: conditional on what we have observed, which parameters maximize the likelihood? The likelihood function is thus:

$$L(\Phi_1, \Phi_2, \tilde{\gamma}, B_2, \Omega; \{\Delta y_t, z_t, c_{t-1}\}_{t=0}^T) = \prod_{t=t_0}^T \frac{1}{2\pi} |\Omega|^{-1/2} \exp \left[-\frac{1}{2} r_t' \Omega^{-1} r_t \right] \quad (5.6)$$

We want to choose our estimates of Φ_1 , Φ_2 , $\tilde{\gamma}$, B_2 and Ω such that the function in (5.6) is maximized. Instead of maximizing L , we can also maximize any monotone transformation of L . It turns out that it is much easier to maximize the log of the likelihood function which is:

$$\ln L(\Phi_1, \Phi_2, \tilde{\gamma}, B_2, \Omega; \{\Delta y_t, z_t, c_{t-1}\}_{t=1}^T) = \sum_{t=1}^T \ln L_t(\Phi_1, \Phi_2, \tilde{\gamma}, B_2, \Omega) \quad (5.7)$$

where

$$\ln L_t = -\frac{1}{2} [2 \ln(2\pi) + \ln |\Omega| + r_t' \Omega^{-1} r_t] \quad (5.8)$$

For the code we will need to provide formulas for some of the properties of $\ln L$. The next step is therefore to find the vector of the first-derivatives (the gradient) and the matrix of the cross and second-derivatives (the Hessian) of the log likelihood function. This is done in Appendix C. The gradient (g) and the Hessian (H) are given in equations (C.9) and (C.11).

5.2 Concentrated likelihood function

In parts of our routine we will also use a *concentrated* likelihood function. The reason will be explained in more detail in Section 5.4, but what we do is to "get rid of" the variance-covariance matrix Ω (hence a more concentrated likelihood function) by implementing the

analytical formula for the ML estimator (MLE) of Ω .

First, let us write the log-likelihood function in (5.7) as:

$$\ln L = -\frac{T}{2} [2\ln(2\pi) + \ln|\Omega| + \text{tr}(\Omega^{-1}W(\Phi_1, \Phi_2, \tilde{\gamma}, B_2))] \quad (5.9)$$

where

$$W(\Phi_1, \Phi_2, \tilde{\gamma}, B_2) = \frac{1}{T} \sum_{t=1}^T r_t r_t' \quad (5.10)$$

and r_t is defined in (5.4). As shown in e.g. Greene (2003, Chapter 14), the MLE of Ω is:

$$\hat{\Omega}_{ML} = W(\hat{\Phi}_1, \hat{\Phi}_2, \hat{\gamma}, \hat{\Pi}_2)$$

where $\hat{\Phi}_1$, $\hat{\Phi}_2$, $\hat{\gamma}$ and $\hat{\Pi}_2$ are the MLEs of these coefficients. Hence, we can impose this in our likelihood function by substituting Ω by $W(\Phi_1, \Phi_2, \tilde{\gamma}, B_2)$ in (5.9). With this we get the concentrated likelihood:

$$\ln L_\beta = -\frac{T}{2} [2(\ln(2\pi) + 1) + \ln|W(\Phi_1, \Phi_2, \tilde{\gamma}, B_2)|] \quad (5.11)$$

We will need the gradient and the Hessian of the concentrated likelihood function as well. We will actually only need to find an approximation to the Hessian – see Appendix C for the argument for this and the derivations. The final results for the gradient (g_β) and the approximated Hessian (H_β) are given in equations (C.42) and (C.43).

5.3 Gamma-concentrated likelihood function

In addition to get rid off Ω we will make the concentrated likelihood function even more concentrated by imposing the analytical MLE of $\tilde{\gamma}$. The reason for this is explained in Section 5.4. Let us first manipulate the last element of g_β into:

$$g_{\beta 4} = \frac{1}{T|W|} (s_{11}s_{32} - s_{21}s_{31} - (s_{11}s_{33} - s_{31}^2)\tilde{\gamma})$$

where s_{ij} is the (i, j) th element of the matrix:

$$S = \begin{pmatrix} \sum_t r_{1t}^2 & \cdot & \cdot \\ \sum_t r_{1t}\hat{r}_{2t} & \sum_t \hat{r}_{2t}^2 & \cdot \\ \sum_t r_{1t}B_2'c_{t-1} & \sum_t \hat{r}_{2t}B_2'c_{t-1} & \sum_t (B_2'c_{t-1})^2 \end{pmatrix} \quad (5.12)$$

with

$$\hat{r}_{2t} = \Phi_2' z_t$$

for $i, j = 1, 2, 3$. The MLE of $\tilde{\gamma}$ is defined as the value setting $g_{\beta 4} = 0$. Hence:

$$\tilde{\gamma}_{ML} = \frac{s_{11}s_{32} - s_{21}s_{31}}{s_{11}s_{33} - s_{31}^2} \quad (5.13)$$

We can impose (5.13) on our system, changing the concentrated likelihood function to be a function of only Φ_1 , Φ_2 and B_2 . Let us write (5.13) as:

$$\tilde{\gamma}_{ML} = \frac{A}{B}$$

where

$$\begin{aligned} A &= s_{11}s_{32} - s_{21}s_{31} \\ B &= s_{11}s_{33} - s_{31}^2 \end{aligned}$$

and then we rewrite $|W|$ as follows:

$$\begin{aligned} |W| &= \frac{1}{T^2} \left(\sum_t r_{1t}^2 \sum_t r_{2t}^2 - \left(\sum_t r_{1t} r_{2t} \right)^2 \right) \\ &= \frac{1}{T^2} (s_{11}s_{22} - s_{21}^2 + \tilde{\gamma}^2 (s_{11}s_{33} - s_{31}^2) - 2\tilde{\gamma}(s_{11}s_{32} - s_{21}s_{31})) \end{aligned}$$

It follows that imposing $\tilde{\gamma} = A/B$ gives the "gamma-concentrated" likelihood function:

$$\ln L_{\beta\gamma} = -\frac{T}{2} [2(\ln(2\pi) + 1) + \ln |W(\Phi_1, \Phi_2, B_2)|] \quad (5.14)$$

with

$$|W| = \frac{1}{T^2} \left(s_{11}s_{22} - s_{21}^2 - \frac{A^2}{B} \right)$$

In Appendix C I derive the gradient ($g_{\beta\gamma}$) and the Hessian ($H_{\beta\gamma}$) – equations (C.54) and (C.59) contain the final results.

5.4 Estimation routine

5.4.1 Regarding maximization and SUR-models in general

Estimating a model with ML will normally involve that the maximum is found numerically. That is: you let the computer choose different values for the coefficients, and based on distinct properties of the likelihood function the computer can follow an algorithm to search for its maximum. The properties the computer will utilize are the gradient and the Hessian, and these

can either be computed numerically or analytically, but the latter requires that you provide the computer with correct formulas. Hence, the simplest code possible to write is one where you only code the likelihood function and then let the computer find the gradient and the Hessian numerically. However, that might cost you both efficiency and precision, especially when the likelihood function becomes complicated. Therefore it is often wise to calculate these properties yourself such that these are found analytically instead of numerically.³²

To complicate things further, it might even be the case – and it will most likely be the case for SUR models – that you will struggle with finding a maximum for the likelihood function (the function does not converge at a point where it is concave), even when you provide a code for both the gradient and the Hessian. To avoid this problem the recommendation from Gould, Pitblado, and Sribney (2003) is to maximize the *concentrated* likelihood function rather than the likelihood function itself. As explained in Section 5.2, the concentrated likelihood function is basically the ordinary likelihood function only that the elements of the variance-covariance matrix of the error-terms (Ω) are concentrated out by using an analytical code for the ML estimates of these elements. This function will be much easier to maximize since you avoid the risk of "getting stuck" in an area with guesses of Ω that are not positive definite.

Still, even fitting a concentrated likelihood function will not solve all problems since the resulting estimates are based on a model where there is no covariance between the estimates of the concentrated model and the estimates of the Ω elements. Hence to get correct standard errors you need to re-estimate the full model (with the standard likelihood function), but now you can use the estimates from the concentrated model as initial values for the maximization process. This will make sure that the computer does not get lost on its way to the maximum, and the resulting estimates get correct standard errors as well.

Furthermore, it will also be such that when you fit the concentrated likelihood, your results might not be robust. By robust I mean that if you re-estimate the model, you will get convergence for the same likelihood value, but slightly different coefficient estimates. Again, the advice from Gould, Pitblado, and Sribney (2003) is to first fit a "constant only" model, which is a model where the only explanatory variable in each equation is a constant. The estimates of the two constants can then be used as initial values when you fit the concentrated model. Giving it these initial values will be enough to secure robust estimates every time.

5.4.2 Regarding maximization and the CCI model

There is one final remark, and that is related to the special-case of estimating this nonlinear SUR model involving a common, unobservable variable fitted with a spline function.

³²In Stata-language a code that only provides the log likelihood function is an lf-code. One that provides the gradient analytically is a d1-code, while one that provides both the gradient and the Hessian analytically is a d2-code.

As we saw in Section 4.2.3, one of the coefficients attached to the CCI-variable had to be normalized to some value to secure identification. Clearly, this normalization should not affect the estimates (except for the scale of CCI and γ). But when I tried to fit either the standard or the concentrated likelihood function, I found that the choice of normalization actually mattered. Changing the normalization changed all the estimates (and it also made the model converge at a different log likelihood value).

This problem is taken care of by the gamma-concentrated likelihood function. By imposing the MLE of $\tilde{\gamma}$ directly (and also change the gradient and the Hessian), the procedure became "balanced" in the sense that the choice of normalization was irrelevant. With this code a change of normalization only changed the scale of the CCI-estimate and the inverse of your previous $\tilde{\gamma}$ estimate was returned.

Knowing that our estimates are unaffected by the normalization is satisfying, but why do we think the problem occurred for the simpler codes? When we inspect our results further it turns out that the gamma-concentrated model actually replicates the estimates from the concentrated model that had the highest log-likelihood value. I.e. one of the normalizations led us to the correct maximum, while the "wrong" normalization caused the code to settle at another local maximum.³³ With the gamma-concentrated model we do not have to worry about choosing the correct normalization.

5.4.3 Strategy for estimation

Based on the discussion in the last subsections, it is clear that we need to write three different codes, and then combine all three of them in a final routine. We write a code for the gamma-concentrated likelihood function, where the gradient and the Hessian are found analytically based on equations (C.54) and (C.59). We write one code for the concentrated likelihood function, where the gradient and the (approximated) Hessian are provided analytically based on equations (C.42) and (C.43). Finally, we write a code for the ordinary likelihood function, where the gradient and the Hessian are provided analytically based on equations (C.9) and (C.11).³⁴ Instead of writing a code for the constant only model, we will use the standard SUR command that Stata provides.

We combine the usage of these three codes in our final routine. This routine is consistent with that recommended by Gould, Pitblado, and Sribney (2003) (except for the gamma-concentrated part, which is our own addition). Section 5.4.1 outlined the general principles we

³³But I have not been able to detect any pattern that can explain which normalization that is the wrong one, ex ante.

³⁴These codes are modified versions of codes provided in Gould, Pitblado, and Sribney (2003). The concentrated code is based on `mysuregc_d2.ado` while the ordinary likelihood code is based on `mysureg_d1.ado` – but both codes have significant extensions. The gamma-concentrated code is a further extension of the concentrated code.

had to follow, and when we include the considerations from Section 5.4.2 as well our routine can be described as:

- Step 1: Use the constant-only model to find initial values for the constants.
- Step 2: Use the initial values calculated in step 1 to fit the model for the concentrated likelihood with $\tilde{\gamma}$ constrained to equal 1 to find initial values for the parameters in Φ_1 , Φ_2 and the constants.³⁵
- Step 3: Use the initial values calculated in Step 2 to fit the model for the gamma-concentrated likelihood function. This code will locate the actual optimum of the likelihood function. Use the results of $\hat{\Phi}_1$, $\hat{\Phi}_2$ and \hat{B}_2 to calculate $\hat{\gamma}_{ML}$ and $\hat{\Omega}_{ML}$. Provide all these values as initial values for the next step.
- Step 4: Use the initial values calculated in Step 3 to fit the model for the ordinary likelihood function. This will produce the final estimates (and the likelihood remains at the optimum located in Step 3). The model should and will converge immediately (but with slightly different coefficient-estimates compared to the gamma-concentrated fitting).

The routine has been written into a Stata command with the name `estimate_cci` such that redoing the estimation and updating the results in the future is very easy. When in Stata, you just type

```
estimate_cci y1 y2 z, cci(d'minyear' d'minyear+1' ... d'maxyear')
```

where `y1` and `y2` are your endogenous variables, `z` contains all the exogenous variables and `d't'` is the sum of all the quarterly shift dummies that belong to year t .³⁶

³⁵This step is not described in any of the preceding sections, but it serves just as a "constant-only" step prior to the fitting of the gamma-concentrated likelihood and it secures "robust" results.

³⁶At present time the code only permits a system with 2 equations – an extension of the code is left for future work.

6 Results

I fit the model in (4.11) using the Stata command `estimate_cci` that I have prepared. Constants and seasonal dummies are included in each equation. Furthermore, to permit some flexibility I choose to "split up" Δy_t by moving Δinc_t over to the right-hand side (and include it as any other variable), leaving the growth of the debt-variables alone on the left-hand side.³⁷

My dataset covers the period 1975Q4-2009Q1 for all the levels variables. Since we need Δ 's and lagged values, the estimation period begins in 1976Q1. The spline function is included as a sum of quarterly shift dummies. Since shift dummies for the same year are assumed to have the same coefficients I add them together to get "smooth" year dummies instead: the dummy for year i takes the value zero prior to year i , the value 0.25 in the first quarter of year i , 0.5 in the second quarter, 0.75 in the third and 1 from the 4th quarter and onward. I include year-dummies for 1977-2008. 1976 is dropped as it would be almost like a constant term.

Let us define a set of **CCI assumptions**. These are based on the information presented in Chapters 2-3 and summarized in Section 4.1.1.

CCI assumptions: Assume that, given the values of *primres* and *addreq*, there is no positive increase in the CCI before 1980. In the period 1984-1987, I assume that the CCI grows the whole time. I also assume that the CCI grows in the period 2004-2007. In 2008 I assume that it decreases. These assumptions translate into sign-restrictions for the dummy-coefficients.

The assumptions reflect the deregulation in the 1980s and also the positive trend in the late 1990s and 2000s as identified by the mortgage survey of Finanstilsynet. Note that I choose not to make any strong assumptions regarding the period covering the banking crisis: I naturally *expect* credit conditions to tighten in this period, but I will leave it to the index to determine this. I also permit the index to drop somewhat in late 90s/early 2000s. This is both motivated by the increase in risk weight for some mortgages in 1998 (see Chapter 2) and the turbulence related to the Asian crisis and the dot-com bubble and bust.

When the system is estimated, the CCI assumptions will be imposed sequentially. First I estimate the system in full generality. Then all the CCI assumptions are checked. Among the coefficients that have violated their sign-restriction, I set the one with the largest p -value (from a one-sided likelihood ratio test) to zero. Next, I estimate the system once more (incorporating the new constraint), and test the remaining assumptions. If none of the coefficients have the wrong sign, I can move on to remove insignificant variables (judged at a 5 % level of significance). For every insignificant variable I drop, I return to the CCI assumptions and check if they are satisfied. All this is repeated until we can go through the CCI assumptions without finding

³⁷When this was done, we subtracted inflation from both growth in debt and income to make them real growth rates.

any violations *and* see that all the remaining coefficients are significant. For a variable to be deemed as insignificant, it must be insignificant both in a partial and in a joint likelihood ratio test. As base for the joint tests I use the log-likelihood value from *the first step after the last time a CCI assumption was violated*.

6.1 Estimates

When the model was estimated, the coefficients for 1978-79, and 2006-08 were all set to zero due to violation of their CCI assumption. However, only that of 2008 was significantly doing so. After this I removed, step-by-step, 27 coefficients. These were all taken out due to lack of significance; at each step I checked if any of the remaining CCI assumptions were violated, but none were. The main guideline for removing insignificant coefficients was to remove the least significant, but this rule was not followed without informed exceptions.³⁸ The estimates are summarized in Table 6.1, and the implied model is:

$$\Delta sd = 0.2418iexp_t - 0.3043\Delta i_t - 0.6653\Delta u_t - 0.1053ecm_{1t-1} \quad (6.1)$$

$$\Delta usd = 0.2258\Delta inc_t - 0.4027\Delta i_t - 0.7021\Delta u_t - 0.5595ecm_{2t-1} \quad (6.2)$$

with the *ecm*-terms given as

$$ecm_1 = \hat{s}d - constant_1 - seasonals_1 - 0.52\hat{h}w - 0.16\hat{m}liq - CCI \quad (6.3)$$

$$ecm_2 = \hat{u}s\hat{d} - constant_2 - seasonals_2 - 0.25\hat{l}iq - 12.99FTB - 3.00CCI \quad (6.4)$$

In the secured debt equation we see that most coefficients have their expected signs. The exception seems to be the coefficient of *iexp*, but there may be ways to partly rationalize this result. One explanation could be that this just illustrates many household's lack of foresightedness – they do not expect interest rates to increase when *iexp* is high, it just happens to be the case that the nominal interest rate is (usually) low when *iexp* is high. Hence, even though *iexp* is the difference between the interest rate on 10-year government bonds and the money market rate, it might end up as an indicator for (minus) the level of interest rates, causing a lower *iexp* to reduce the growth in secured debt. That *iexp* is normally high when *i* is low is supported by Figure 6.1, especially in the latter part of the period. Hence, we are getting an interest rate effect in through the back door, not a weird expectations effect.³⁹

³⁸If it had been, then $[\Delta sd]incexp$, $[\Delta usd]\hat{h}w$ and the spline-dummy of 1989 would still have been in the model while $[\Delta usd]\hat{l}iq$ would have been dropped. These alternative models are not nested, so there are no formal ways of testing the validity of one relative to the other.

³⁹For those who do not want to reject this as an expectations effect it might be argued that the coefficient is positive because households increase their stock of debt when they expect rates to increase. They do so because they know that it will be harder for them to get accepted by the bank once the cost of servicing a loan increases. However, I do not find this argument too compelling.

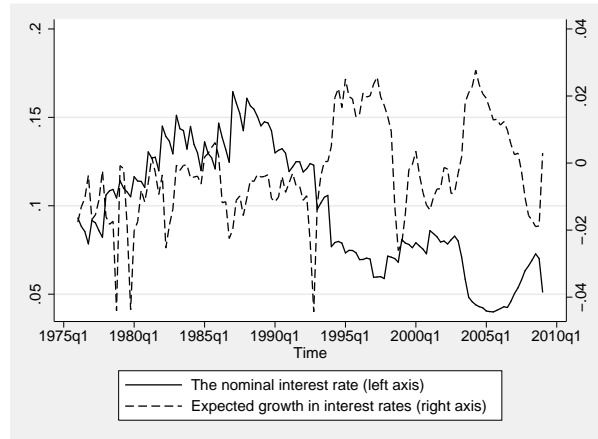


Figure 6.1: The interest rate and the expected growth in rates

Looking at the remaining coefficients in the sd equation we see that growth in nominal interest rates appears to have the negative cash-flow effect we expected. An increase in the unemployment rate also has a negative impact. This captures both the increased uncertainty and potentially also some income shock effects. Housing wealth and moderately liquid wealth (to income) has the expected effects with elasticities of 0.52 and 0.16, respectively. The speed of adjustment coefficient is 0.11, implying that it will take a little more than 2.5 years for secured debt to return from a 1 percentage deviation from the estimated long-run equilibrium.

For unsecured debt we find that higher income growth increases the growth in unsecured debt. A negative impact comes from growth in interest rates and unemployment. Note that these coefficients are quite similar to those for secured debt. Liquid wealth ends up with a positive effect with an elasticity of 0.25. It implies that having additional security can lead to a larger stock of unsecured debt. The proportion of the population aged 20-39 relative to those aged 20-74 has a *very* strong and positive effect, with a semi-elasticity of 12.99. The sign seems correct, but the size of this coefficient is probably too large. This result is discussed further in Section 6.1.4. The CCI is estimated to have a relatively stronger impact on unsecured than secured debt. This means that unsecured debt responds stronger to changes in the credit conditions than secured debt, which is reasonable, but this might be too strong and it is therefore also discussed more in Section 6.1.4. The speed of adjustment is faster for unsecured than secured debt. This fits well with the fact that unsecured debt is more short-term than secured. Judged by the R^2 statistics for the two equations, the model seems to capture about 3/4 of the variation in the growth of the two debt variables, but remember that the R^2 statistic has a less straightforward interpretation in a SUR model since the errors are correlated.

Based on the estimates of Ω it seems like the stochastic noise in the growth of unsecured debt is more volatile than that in the growth of secured, and this too seems reasonable. The covariance term is in principle not significant, but we have kept it in the model despite this.

Table 6.1: Maximum likelihood estimates of the model in (4.11)

Variable	Δsd_t			Δusd_t		
	Coef.	Std.Err	<i>p</i> -value ^a	Coef.	Std.Err	<i>p</i> -value ^a
$iexp_t$.2418	.0733	.0012	(dropped)		
Δi_t	-.3043	.1069	.0051	-.4027	.1711	.0200
Δu_t	-.6653	.2220	.0032	-.7021	.2950	.0189
Δinc_t	(dropped)			.2258	.0690	.0012
\hat{hw}_{t-1}	.0548	.0068	.0000	(dropped)		
\hat{liq}_{t-1}	(dropped)			.1374	.0596	.0233
\hat{mliq}_{t-1}	.0168	.0070	.0188	(dropped)		
FTB_{t-1}	(dropped)			7.2680	1.1902	.0000
$B'_2 c_{t-1}$	1			15.9577	4.4427	.0003
\hat{sd}_{t-1}	-.1053	.0153	.0000	(dropped)		
\hat{usd}_{t-1}	(dropped)			-.5595	.0740	.0000
Dummy for quarter 2	(dropped)			.0082	.0034	.0181
Dummy for quarter 4	.0062	.0027	.0210	(dropped)		
Constant	.0283	.0120	.0189	-3.4571	.5528	.0000
σ_{11}	.0001076	.0000132	.0000			
σ_{12}	-.0000184	.0000156	0.240			
σ_{22}	.0002533	.0000313	.0000			
R^2	.7544			.7328		
Log likelihood	781.67392					
No. of obs.	133					

^a*p*-values are based on LR-tests, except for those of the σ 's and the CCI coefficient of Δusd which are based on Wald tests.

6.1.1 Cointegration

Taking the long-run coefficients as given, do we find evidence of cointegration? For the variables entering the long-run relationships of secured and unsecured debt to income to cointegrate, the error-correction terms must be stationary. This can be tested using ADF tests as outlined in Appendix B. At a 5 % level of significance I find, unfortunately, that only the error-correction term of the unsecured debt equation is stationary (*t*-value of -3.587 with 4 lags), while that of secured debt ends up with a *t*-value of -2.369 (5 lags). However, low power of the ADF tests is always a problem, making it important to inspect the error-correction terms visually as well. Figure 6.2 shows the two series. Inspecting the series does not make us surprised that the upper series fails to end up as stationary, but at the same time it does seem like we have detected something close to a reasonable long-run relationship. ecm_1 starts fluctuating at around .01 until, first the boom in the late 1980s (leading secured debt over income to exceed its long-run value) and the bust at the time of the banking crisis. However, from the 2000s it

seems like it begins fluctuating around a new level (.02) until it drops again around 2008. For unsecured debt we see a sharp drop in the early 1980s (caused by the drop in the CCI and also consistent with how we interpreted this drop, see Section 6.1.3), and then a boom through the 80s. When the banking crisis kicks in, the error-term for unsecured debt drops as well, and afterwards there are no huge deviations from its long-run equilibrium. In total my judgement is that despite lack of formal evidence, it does seem like we have something close to cointegration in the *sd* equation. Formally I detect cointegration for the *usd* equation, but here we do have some weird coefficient estimates that confuse the picture.

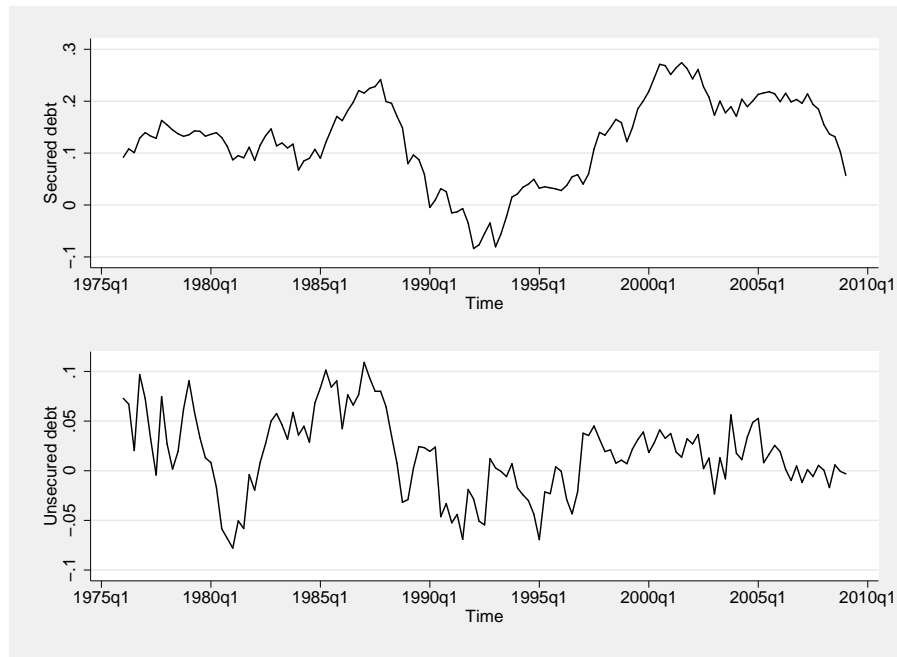


Figure 6.2: The error-correction terms.

6.1.2 Diagnostics

Another important criterion for evaluating a model is whether data supports the assumption that the errors in the model are uncorrelated over time and bi-normally distributed. I can make my job much simpler if I re-run the model with the restriction $\sigma_{12} = 0$ to avoid the *bi*-normal distribution when I inspect the residuals. This passes (*p*-value of 0.2367 from the likelihood ratio test) and it does not change the estimates much. The resulting residuals can then be tested using single-equation methods.

Let us first perform a visual diagnostic test. Figure 6.3 shows how the residuals are distributed and also their autocorrelation. In both histograms I have added a normal density function with the first two moments equal to the empirical ones for the residual to make the judgement simpler. For both residuals it looks like assuming a normal distribution is a quite

good approximation. It seems like autocorrelation is not an issue for the secured debt equation, but potentially for that of unsecured debt.

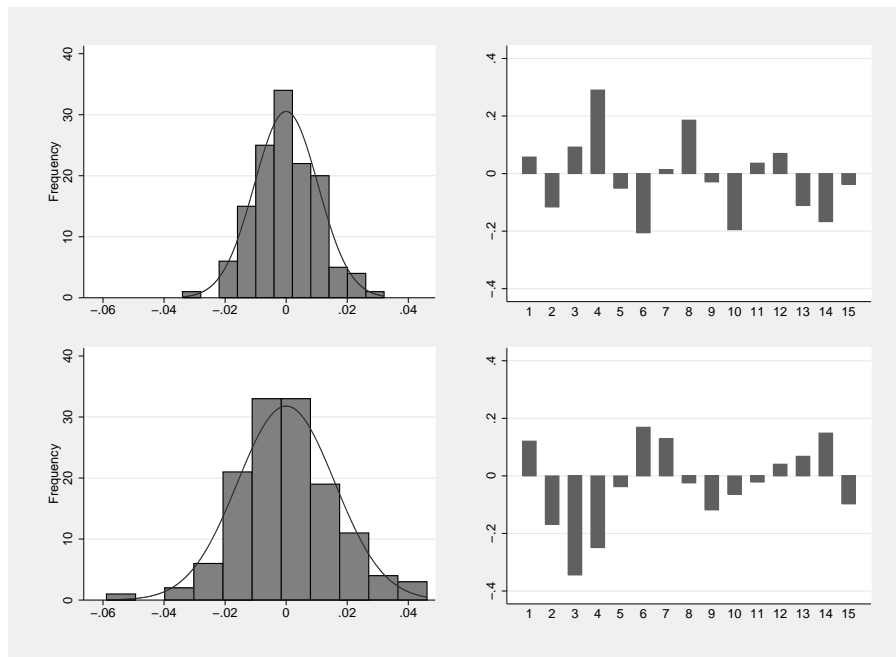


Figure 6.3: Visual diagnostics for residuals (top panels are for the residuals from the sd-equation, bottom for the usd-equation). Their frequency (left panels) and autocorrelograms (right panels).

Secondly we can perform more formal diagnostic tests. To test for autocorrelation I run Breusch-Godfrey Lagrange multiplier tests (see description in Greene (2003, Chapter 12)). When testing for autocorrelation of order p for residual i , \hat{e}_i , the test statistic is $LM = TR^2$, with R^2 defined as the standard statistic from the regression

$$\hat{e}_{it} = \beta' x_{it} + \sum_{j=1}^p \rho_j \hat{e}_{i,t-j} \quad (6.5)$$

where x_{it} is the vector of variables from the final form of equation $i = 1, 2$. The missing values for the lagged residuals are replaced by zeros in the regression. As LM is asymptotically distributed $\chi^2(p)$, this is a test it is easy to implement.

Normality is tested for by conducting two different tests. The first combines tests on the skewness and on the kurtosis, as suggested by D'Agostino, Balanger, and D'Agostino Jr. (1990) and implemented as the command `sktest` in Stata. The second is a Shapiro-Wilk test for normality due to Shapiro and Wilk (1965), implemented as the command `swilk` in Stata.

Table 6.2 gives the results, and none of the p -values give evidence for rejecting the null-hypothesis of no autocorrelation. Neither can normality be rejected in any of the two tests. Hence, based on the residuals, it seems like the model is well-specified.

Table 6.2: Diagnostics

	Normality		Autocorrelation	
	<i>p</i> -value (sktest)	<i>p</i> -value (swilk)	Lags	<i>p</i> -value
\hat{e}_1	.451	.8367	1	.6814
			2	.9192
			3	.9825
			4	.9966
			5	.9994
\hat{e}_2	.0809	.1715	1	.7918
			2	.9658
			3	.9952
			4	.9994
			5	.9999

6.1.3 The estimated CCI

As already noted, the dummies for 1978, 1979, 2006 and 2008 were set to zero due to violation of their CCI assumption. Furthermore, I dropped the dummies for 1977, 1983, 1989-90, 1992-93, 1995-96, 1998 and 2002 due to their lack of significance. *primres* and *addreq* were also dropped due to insignificance. The estimates are summarized in Table 6.3 and the CCI is graphed in Figure 6.4. Please remember that an increase in CCI should be interpreted as an easing of credit conditions.



Figure 6.4: The estimated CCI, 1975Q4-2008Q4.

The shape of the index is close to what I hoped for, given the qualitative information summarized in earlier chapters, even though it begins with a quite sharp contraction in 1980-81. However from 1982 it increases rapidly until its peak in the end of 1987. The first drop in the index seems a bit peculiar given what we know about the institutional development. But

the dip could capture that this was a period of strong growth in the *grey market*, making the amount of "white" debt increase less than what the fundamentals implied, causing the index to fall. Furthermore, there might also be lagged effects from the contractive policies at the end of the 1970s. The increase in 1982-87 is easier to interpret: this clearly captures the effects of the deregulation, leaving fewer households credit constrained.

The index drops in 1988-94, corresponding to the period of the banking crisis. This too seems reasonable, even though some might claim that the drop should have been even greater. At the bottom in 1994 the index is back to its mid-1986 level. From 1995 and onwards there are no other negative reversals in the index and it increases quite steadily. By 1999 the index is back to its 1987 level, and ends at more than double this level in 2005. This increase probably reflects a reduction in credit constraints not driven by deregulation, but by financial innovation and the introduction of such instruments as home equity credit lines, and also potentially the internationalization of capital flows and availability of credit for Norwegian banks. Increased competition in the banking sector can also have been a factor. However, it might be that the index grows too much between 1997 and 2005: one potential reason is discussed in Section 6.1.4.

Table 6.3: CCI estimates (B'_2)

Sum of qd 's for:	Coef.	Std.Error	p -value ^a
1980	-.0067	.0018	.0000
1981	-.0056	.0017	.0003
1982	.0045	.0014	.0000
1984	.0062	.0017	.0000
1985	.0057	.0016	.0000
1986	.0068	.0019	.0000
1987	.0071	.0019	.0000
1988	-.0026	.0010	.0004
1991	-.0047	.0012	.0000
1994	-.0020	.0007	.0005
1997	.0060	.0015	.0000
1999	.0039	.0012	.0001
2000	.0040	.0013	.0005
2001	.0036	.0012	.0005
2003	.0044	.0013	.0000
2004	.0046	.0014	.0001
2005	.0028	.0011	.0057

^a p -values are based on LR-tests

6.1.4 Reasonable and relevant results?

I have presented a model which, with a few noted exceptions, has long-run coefficients that are plausible, residuals that seem to be normally distributed and without autocorrelation, and an implied credit conditions index that has a shape that seems to capture most of what I expected it to. But getting flawless results would be "too good to be true", and there are of course signs of weaknesses left. In particular, I do not find strong evidence of cointegration in the equation for secured debt, and in that of unsecured debt I get at least two quite unreasonable coefficients (that of *FTB* and partly that of *CCI*). Furthermore the *CCI* potentially grows too much during the last part of our sample.

I suspect that these problems are in large part the symptom of omitting one important mechanism: the potential for substitution between secured and unsecured debt. As secured debt is normally cheaper than unsecured debt, secured debt will be preferred to unsecured. Thus an agent will never increase his amount of unsecured debt if he has the possibility of taking up secured debt instead. We may call this *uni-directional substitution*. However, there is no reason to expect this to be a central mechanism in a credit-rationed regime, such as Norway in the early 1980s. But as the credit markets were liberalized this mechanism should become more important, especially when home equity credit lines made secured credit even more available during the 2000s. For instance, if an agent needed credit in order to purchase a new car in the early 1990s it was probably necessary to take up a car loan. On the contrary, in 2008 the agent might just draw on his/her home equity credit line instead.

This implies that there might be *two separate effects* on the amount of both secured and unsecured debt from financial liberalization. First we have the direct effect: more liberalized credit markets makes credit more available. This corresponds to what we have modelled as the *CCI*-effect. Second we have the indirect effect: in a more deregulated regime the degree of uni-directional substitution might increase, and households switch from unsecured to secured credit. This is an effect that is left out in our model, but we would expect it to be a mechanism that starts working some time in the 1990s.

If this really is the problem, how should it have affected my estimates? In the secured debt equation, the positive boost from the substitution may have caused one of the wealth variables to have a too large effect. It might also have caused a positive push to the credit conditions index, causing it to increase more during the last third of our period than what it "structurally" should have done. What about the equation for unsecured debt? In lack of the substitution effect, we should have seen one of the other variables causing a too negative impact after the mid-1990s. Conveniently, it might actually be that parts of this negative trend is captured in the coefficient of *FTB*. As we see in Figure B.4, *FTB* starts declining in the mid 1980s, and drops at a steady rate after around 1995. Hence, by giving *FTB* an unreasonably large coefficient, parts of the substitution effect is taken account of. Furthermore, since *FTB* starts dropping a

bit too early to act as the "perfect trend", this might also cause the relative CCI coefficient to be too large in order to counter-act the negative impulses from *FTB* in the late 80s. This can also have pushed the CCI to fall too little during the banking crisis.

My preliminary conclusion is therefore that parts of the large coefficient on both *FTB* and *CCI* in the unsecured debt equation, and also the possibly too strong growth of the CCI in the end of the period, reflect that households are substituting from unsecured to secured debt, reducing the long-run level of unsecured debt and correspondingly increasing that of secured debt. This could also explain why the error-correction term of secured debt seems to settle at a higher level at the end of the 90s while the error-correction term of unsecured debt stays at a low level after the banking crisis. This would also explain why I do not find striking evidence for cointegration, but at the same time why it seems like I have detected relationships that make some sense.

6.2 Comparison with results from other studies

As already noted, Fernandez-Corugedo and Muellbauer (2006) estimate a 10 equation system, where two of the equations are equations for the log of secured and unsecured debt. The long-run relationship they find, using UK data for the period of 1976Q1-2004Q4, are (found in Tables A1 and A2):

$$\hat{s}d = 0.59inc_{cap} - 1.81\hat{l}i\hat{q} + 0.07i\hat{l}l\hat{i}q + 0.29\hat{h}w + 3.7DEM + 4.68CCI \quad (6.6)$$

$$u\hat{s}d = 0.59inc_{cap} - 0.29\hat{l}i\hat{q} + 0.07i\hat{l}l\hat{i}q - 0.11\hat{h}w + 0.18cred + 2.08CCI \quad (6.7)$$

where $\hat{x} = \log(X/INC)$, inc_{cap} is the real income per capita, $ILLIQ$ is defined approximately as the sum of our variables $MLIQ$ and $ILLIQ$, DEM is the share of people aged 20-34 and $cred$ is the log ratio of the number of credit cards relative to the adult population. In addition these equations include various interest rate effects and they also contain some risk-terms (involving among other things the volatility of interest rates and inflation). The implied CCI was given in Figure 1.1 (labeled "No interaction") above.

Compared to my results we see that they find a positive effect from income per capita. The elasticities of housing wealth and illiquid wealth for secured debt are in the same region as mine. For unsecured debt they get an opposite effect of liquid wealth, but my estimate might capture some of the positive impact from illiquid wealth as well. Contrary to my results, they find that the CCI has more than twice as large impact on secured debt to income relative to unsecured debt.

A Norwegian study it is relevant for us to compare my results with is Jacobsen and Naug (2004). They estimate an equation for household's total stock of debt using data for the period of 1994Q1-2004Q4. They do not control for any changes in the credit conditions, but

comparing our estimates with theirs is still of interest. Their long-run relationship is:

$$debt = hw - 1.7i + 0.17turnover + 0.64student \quad (6.8)$$

where *debt* is approximately the log of $SD + USD$, *hw* and *i* are defined as we do (using slightly different sources), *turnover* is a variable measuring the number of trades made in the housing market during a quarter and *student* is the share of students aged 20-24 relative to the rest of the population.

This study supports our result that some form of a demographic variable should be included, and also that our variable probably gets a too large coefficient. They find a much stronger housing wealth effect than us, but they ignore the other wealth effects that appear in our model.

In a study of bank lending and property prices in Hong Kong, Gerlach and Peng (2005) find that the logs of real bank lending (*credit*), real GDP (*gdp*) and real property prices (*hp*) cointegrate and that it is bank lending that error-corrects. Their long-run equation is:

$$credit = gdp + 0.36hp \quad (6.9)$$

In their dynamic model for bank lending, they also show how a regulatory change in 1991 (when banks in Hong Kong started to apply stricter loan-to-value limits) has a significantly negative impact on how strong house price changes affects the growth in credit. However, this is not taken into account in the long-run analysis.

Hofmann (2004) has much of the same starting-point as Gerlach and Peng (2005), but considers the evidence for 16 different countries. Two different long-run relationships are posited, one where the log of credit is determined by the log of GDP and the real rate of interest, and one where the log of property prices is added as well. Hofmann finds that the specifications ignoring property prices fail to detect cointegration in 11 out of 16 countries, but the one that includes property prices detects cointegration in 15 out of 16 countries. The relationships vary across countries, and the long-run equation for Norway is:

$$credit = 2.369gdp - 0.077r + 3.828hp \quad (6.10)$$

Both these studies indicate that income and house prices are central determinants for the stock of debt, but the coefficients of Hofmann (2004) are much bigger than those implied by our model.

7 Conclusions

This thesis has provided an estimate of a credit conditions index (CCI) for Norway, based on the framework and methodology of Fernandez-Corugedo and Muellbauer (2006). The index has a shape close to our *ex ante* expectations, even though the model used to produce it is plagued with some problems. There is no reason to believe that this is "the" index, but it should neither be dismissed as irrelevant. In lack of better measures, this index provides a fine opportunity to control for the institutional changes that have occurred since the mid 1970s. It is likely that ignoring such changes is worse than applying an imperfect index.

As important as the estimates on their own are the steps that were taken prior to estimation. This thesis has documented the institutional development in a detailed fashion, making this knowledge more available than what it has been earlier. Having some idea of how the deregulation took place is essential for other researchers that are to perform studies related to the Norwegian credit markets, and Chapter 2 can therefore serve some useful purposes. Furthermore, the maximum likelihood framework that is used to estimate the model is documented quite extensively. If future researchers want to use model formulations that are difficult to implement using the nonlinear SUR commands that are available in standard statistical software, this documentation can serve as a good starting point if they are to write their own maximum likelihood codes.

In my opinion, future research related to the topic of credit condition indices can follow two alternative strategies. The first is to stay within the framework of Fernandez-Corugedo and Muellbauer (2006), which is what I attempted to in this thesis. This strategy, as I interpret it, entails that you use indicators for the credit conditions that are as "pure" as possible in order to support the argument that you detect a true structural trend. An ideal set of indicators mixes both micro and macro observations and the indicators should also preferably be based on variables that are not of particular interest in other applications. The various loan-to-value and loan-to-income ratios constructed by Fernandez-Corugedo and Muellbauer (2006) are perfect examples. The estimated CCI will then, if results come out as intended, represent the common structural trend in these variables and can be used directly in other applications as an ordinary variable.

What I interpret as the second strategy is more closely related to parts of Blake and Muellbauer (2009). This study does produce CCI estimates, but the CCI methodology is mainly being used to control for the credit conditions in an analysis where the main purpose is to estimate equations for house prices and the stock of debt. Hence, instead of first estimating a CCI and then apply it in a model, the two things are done simultaneously. This reduces the structural interpretation of the CCI somewhat, and it might be difficult to estimate equations for your left-hand side variables that are satisfactory if the purpose is to e.g. produce forecasts

for these variables. However, it might be argued that a simple model could be used to estimate the CCI as a first step, and then a second step could condition on the estimated CCI and use it in a model with more flexible dynamics. Alternatively one could pursue a strategy similar to that of Williams (2009) and use a stochastic trend to determine where the important break points of the CCI are. But this hinges on the possibility of including a *common* stochastic trend in all the equations of your system, something as far as I am aware of is not possible in any statistical software at present time.

My primary suggestion for future work is to stay within the main framework applied in this thesis, at least at first. The most pressing challenge is to think of a way to incorporate the shift from unsecured to secured debt that I believe has happened. One should also search for other potential credit indicators. If this does not succeed, the second strategy should also be given a try. A way to expand the analysis could be to include an equation for aggregate consumption and let the CCI be interacted with e.g. wealth variables or the interest rate. Another alternative could be a house price equation where the income-response is interacted with the CCI. Even if the attempts with the first strategy are successful, the second strategy could be a useful exercise to evaluate the robustness of the CCI estimates.

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A Documentation of the data

A.1 Method for extracting a seasonal pattern

First, let us look at how one can construct quarterly series for a variable even though you only have annual observations of it. To do so you need quarterly observations of another variable that you think has a similar seasonal pattern.

Let us consider the variable X . Let X_t be the value of X at time t (the time interval is quarters). For the period $t = 0, 1, 2, \dots, T$, we have observed $X_{\tilde{t}}$ for $\tilde{t} = 0, 4, 8, \dots, T$ (assuming T refers to a fourth quarter). We also have data for a variable Y . For this variable we have full quarterly observations. We believe that the seasonal pattern of Y is quite close to that of X . Is there any way to use Y to construct quarterly observations of X ?

I propose a method where we force X to have the same quarterly growth rate as Y , only adjusted for a shift-component α_i unique for each year i . If X and Y have identical values both at the end of year $i - 1$ and i , we want $\alpha_i = 0$. If X grows and Y drops during year i we want $\alpha_i > 0$ and vice versa. Let p_t be the growth rate of Y in period t . Given the value of α_i we construct quarterly observations according to the formula $X_t = (1 + p_t + \alpha_i)X_{t-1}$ for $t = z_i + 1, z_i + 2, z_i + 3$, where z_i is the constant such that the values of t belong to year i . Since we want the numbers to "add up" within every year, the shift-component α_i is implicitly defined by:

$$\frac{X_{z_i+4}}{X_{z_i}} = (1 + p_{z_i+1} + \alpha_i)(1 + p_{z_i+2} + \alpha_i)(1 + p_{z_i+3} + \alpha_i)(1 + p_{z_i+4} + \alpha_i) \quad (\text{A.1})$$

for any value i that refers to a year in the set of observations.

For practical purposes, solving this non-linear equation for every year is a bit time consuming, so it might be better to use an approximation:⁴⁰

$$\alpha_i \approx \frac{\left(\frac{X_{z_i+4}}{X_{z_i}} - (1 + p^{i0}) - p^{i1}p_{z_i+4} - p^{i2}p_{z_i+3} - p^{i3}p_{z_i+2} \right)}{4 + 3p^i} \quad (\text{A.2})$$

where

$$p^{ij} = \sum_{t=z_i+1}^{z_i+4-j} p_t$$

for $j = 0, 1, 2, 3$.⁴¹

⁴⁰This approximation is somewhat arbitrary as it simply cuts off some of the remaining terms, but these terms will never be of any great size.

⁴¹Note that you should not "regenerate" X in the fourth quarter if α_i is approximated.

A.2 Constructing series for secured and unsecured debt

Our main source⁴² (MS) is a dataset which gives us series for total loans to households and total loans for housing purposes to households from commercial banks, savings banks, the Post Office Savings Bank, loan associations and state banks.⁴³ For the two first institutions we have yearly observations for 1975-1987, and monthly thereafter. For the Post Office Savings Bank the monthly data starts in 1993, and for the latter two in 1996.

To supplement the main source we have one supplementary source⁴⁴ (SS) which has quarterly observations for 1975Q1-2009Q1 for total loans to household from commercial banks, savings banks, the Post Office Savings Bank, loan associations, state banks, financing companies, life insurance companies⁴⁵ and non-life insurance companies.

To get quarterly series for secured debt (*SD*) and unsecured debt (*USD*) we go through the following steps:

1. Get quarterly observations for total loans from commercial banks, savings banks, the Post Office Savings Bank, loan associations and state banks from MS in the period such observations lack by applying the method described in Section A.1 with the equivalent series from SS as a basis for the seasonal pattern.
2. Get full series for total loans from financing companies, life insurance companies and non-life insurance companies by adopting the entire series from SS.
3. Get quarterly observations for loans for housing purposes from commercial banks, savings banks, the Post Office Savings Bank, loan associations and state banks from MS in the period such observations lack by applying the method described in Section A.1 with the series on total loans from the respective institutions in MS as a basis for the seasonal pattern.
4. Get series for secured and unsecured debt. Make the assumption that all loans for housing purposes are secured, and that all remaining loans are unsecured. For financing companies, life insurance companies and non-life insurance companies, we do not have any division between loans and housing loans. Here we assume that all loans from financing

⁴²Source: Statistics Norway

⁴³For state banks the series from MS contains an error in the period of 1990-1994 as it suddenly shifts down by 25 percent and then up again in 1995Q1. Luckily, the series from the supplementary source is correct, and we'll use these observations in this period (the series match perfectly prior to 1990 and after 1994). To correct the housing loan series we add an element in every year (this is done before we obtain quarterly observations) such that housing loans' share of total loans remains the same as before the correction.

⁴⁴Source: Norges Bank's database FINDATR. The data series end in 2003, and the observations after that are appended from the database which succeeded FINDATR, FINSE (produced at Statistics Norway).

⁴⁵When we append the FINSE numbers, we add together life insurance companies, pension funds, bonds and certificates to get the whole category 'life insurance' consistent with FINDATR.

companies are unsecured, and that all loans from life and non-life insurance companies are secured.

The first step is uncontroversial, but the last steps might be less so. I admit that this is not producing perfect series, and that some of the remaining "unsecured debt" is potentially secured. But it is much better than nothing. Remember also that it is only the seasonal pattern that gets a little bit wrong, the level is correct through the whole series since we always have at least one observation in every year.

A.3 Other variables

Most other variables have been extracted from the database of Statistics Norway without any modifications. *INC* is given as household's net income (RD300 from Statistics Norway) minus dividends (RAM300). *HW* is defined as the price per unit of housing capital (PBOL) times households' stock of housing (K83). *i* is 4 times the average quarterly interest rate households are facing (RENPF300). *r* is *i* times $1 - \tau$ minus the yearly inflation rate where τ is the capital tax rate (TRTMNW). The inflation rate is based on the Norwegian CPI (KPI). *u* is the unemployment rate (URKORR). *FTB* is based on various population numbers from Statistics Norway.

To get *iexp* I use the difference between the interest rate on Norwegian 10-year government bonds and the money market rate. Data on the interest rate on Norwegian 10-year government bonds from 1985Q1-2009Q1 and on the money market rate (3 month NIBOR) from 1978Q3-2009Q1 were extracted from the web site of Norges Bank (www.norges-bank.no). Data on the bond rate prior to 1985 was found in Eitrheim, Klovland, and Qvigstad (2004, Chapter 4, Table A4). Household's average interest rate is used as a proxy for the money market rate prior to 1978Q3.

Data for *LIQ* (households' holdings of notes, coins and deposits) and *MLIQ* (households' holdings of bonds, stocks, loans and other claims) are only available from Statistics Norway after 1995. To get data prior to 1995 I combine this source with similar data from FINDATR, Norges Bank's old data base (updated until 2002/3 but no longer). The levels do not match perfectly between these sources, so in order to combine them I use the growth rates from the FINDATR data to create series prior to 1995.

A.3.1 Constructing income expectations

To construct a variable for income expectations I follow a setup similar to that of Muellbauer and Murphy (1993), also applied in Aron, Muellbauer, and Murphy (2008). I assume that households in each period use the *H* latest observations of income to forecast their income the next *k* quarters and based on this they construct a measure for the deviation between their

permanent and *current* income. Formally I assume that the log deviation between household's (expected) permanent income (INC^p) and current income (INC) is given by:

$$iexp_t = inc_t^p - inc_t = \frac{1}{\sum_{i=1}^k a_i} \sum_{i=1}^k a_i \hat{E}_t \Delta inc_{t+i} \quad (A.3)$$

where k represents the horizon of the agent, a_i are the (possibly varying) discount factors, and \hat{E}_t is the 'estimations operator'. As in Aron, Muellbauer, and Murphy (2008), I will set $k = 12$ and $a_i = 0.85$. H is set to 32 – implying 8 years of memory.⁴⁶ To get estimates for future income, I assume that the households in every period t^* use the H latest observations of income to perform the regression:

$$\Delta inc_t = \alpha_0^{t^*} + \sum_{i=1}^5 \gamma_i^{t^*} \Delta inc_{t-i} + \varepsilon_t \quad (A.4)$$

for $t^* = t_0, t_1, \dots, T$. Based on these results they "forecast" future income growth:

$$\hat{E}_t \Delta inc_{t+1} = \hat{\alpha}_0^t + \sum_{i=1}^5 \hat{\gamma}_i^t \Delta inc_{t+1-i} \quad (A.5)$$

$$\hat{E}_t \Delta inc_{t+2} = \hat{\alpha}_0^t + \hat{\gamma}_1^t \hat{E}_t \Delta inc_{t+1} + \sum_{i=2}^5 \hat{\gamma}_i^t \Delta inc_{t+2-i} \quad (A.6)$$

$$\hat{E}_t \Delta inc_{t+3} = \hat{\alpha}_0^t + \sum_{i=1}^2 \hat{\gamma}_i^t \hat{E}_t \Delta inc_{t+3-i} + \sum_{i=3}^5 \hat{\gamma}_i^t \Delta inc_{t+3-i} \quad (A.7)$$

$$\hat{E}_t \Delta inc_{t+4} = \hat{\alpha}_0^t + \sum_{i=1}^3 \hat{\gamma}_i^t \hat{E}_t \Delta inc_{t+4-i} + \sum_{i=4}^5 \hat{\gamma}_i^t \Delta inc_{t+4-i} \quad (A.8)$$

$$\hat{E}_t \Delta inc_{t+5} = \hat{\alpha}_0^t + \sum_{i=1}^4 \hat{\gamma}_i^t \hat{E}_t \Delta inc_{t+5-i} + \hat{\gamma}_5^t \Delta inc_t \quad (A.9)$$

$$\hat{E}_t \Delta inc_{t+j} = \hat{\alpha}_0^t + \sum_{i=1}^5 \hat{\gamma}_i^t \hat{E}_t \Delta inc_{t+j-1} \quad (A.10)$$

for $j = 6, \dots, 12$.

⁴⁶With income series starting in 1967Q1, the series for $inc_t^p - inc_t$ will start in 1975Q1.

B Tests for stationarity

I apply standard augmented Dickey-Fuller tests to check the degree of integration in the variables.

B.1 Basic theory

Consider a time series variable Y_t which I assume follows the process

$$Y_t = \rho Y_{t-1} + \varepsilon_t \quad (\text{B.1})$$

A time series Z_t is said to be *stationary* if its distribution has the properties (see definition in Hamilton (1994, Chapter 3)):

1. $E(Z_t) = \mu$ for all t
2. $Cov(Z_t, Z_{t-j}) = \gamma_j$ for all t and all values of j

These requirements only seem reasonable for a few macroeconomic variables. Most macro variables are growing over time such that we would neither expect the mean nor the variance of these series to remain constant. Such series are labeled non-stationary. The standard terminology is that a stationary variable is integrated of order 0, giving them the name $I(0)$ variables. A non-stationary variable is integrated of order $d \geq 1$ and is called an $I(d)$ variable. The order of integration d is determined by the number of times the variable must be differenced in order to make it become stationary. The most common type is $I(1)$ variables. These become stationary after being differenced once.

What can we say about the the order of integration of Y_t ? Let us write (B.1) as:

$$Y_t = \rho^j Y_{t-j} + \sum_{i=0}^j \rho^i \varepsilon_{t-i} \quad (\text{B.2})$$

Assume first that $|\rho| < 1$ and let j go to infinity. We see that this makes Y_t a stationary variable since its distribution then has the properties:

1. $E(Y_t) = 0$
2. $Cov(Y_t, Y_{t-j}) = \frac{\rho^j}{1-\rho^2} \sigma^2$

which fulfill the requirements for begin stationary. Hence, $|\rho| < 1$ implies that Y_t is an $I(0)$ variable.

Assume next that $\rho = 1$. From (B.2) it is clear that the variance of Y_t will be infinite implying that it cannot be stationary. Rewriting (B.1) we see that $\Delta Y_t = \varepsilon_t$. The distribution of ΔY_t has the properties:

1. $E(\Delta Y_t) = 0$
2. $Var(\Delta Y_t) = \sigma^2$
3. $Cov(\Delta Y_t, Y_{t-j}) = 0$ for $j \neq 0$

making it a stationary variable. Hence, $\rho = 1$ implies that Y_t is an I(1) variable.

B.2 Testing for stationarity

Suppose we have T observations of Y_t . Motivated by the results in the previous section, it seems like a good idea to test for stationarity by regressing

$$\Delta Y_t = \alpha Y_{t-1} + \varepsilon_t \quad (\text{B.3})$$

for $t = 1, \dots, T$, and then test whether the estimate of α is significantly less than zero. As $\alpha = \rho - 1$, this is the same as testing if the estimate of ρ is significantly less than 1. We will estimate (B.3) as usual using OLS, but we cannot use ordinary t-tests because if $\rho = 1$, the asymptotic distribution of the OLS estimate is non-standard (see Hamilton (1994, Chapter 17)). We can still use t-values as our test statistic, but to get the correct distribution of the t-values we have to use the *Dickey-Fuller distribution*. This test is a so-called Dickey-Fuller test.

Furthermore, to allow for the error term ε_t to be autocorrelated and also to allow for a time trend and a constant in (B.3) we will rather estimate the equation (see Hamilton (1994, Chapter 17))

$$\Delta Y_t = \mu + \beta t + \alpha Y_{t-1} + \sum_{i=1}^k \alpha_i \Delta Y_{t-i} + \varepsilon_t \quad (\text{B.4})$$

where k is chosen by the researcher to make sure that the resulting residual is without autocorrelation.⁴⁷ This is a so-called augmented Dickey-Fuller (ADF) test.

Our procedure for testing stationarity will be the following:

1. Estimate (B.4):
 - (i) Estimate (B.4) with $k = 5$
 - (ii) Check if β is significant (using an ordinary t-test), if not: drop it
 - (iii) Estimate (B.4) (possibly without a trend) again 6 different times (with $k = 0, 1, \dots, 5$).⁴⁸

⁴⁷This implies that you model Y_t as an AR($k+1$) process.

⁴⁸Note that the selection of k will involve a slight inconsistency since when we reduce the number of lags by one, we get one extra observation as well. One could argue that the same number of observations should be used when testing for all the lags. However, we do give the lag length with the smallest number of observations priority by testing it first, and this should do more than enough to compensate.

(iv) Choose the highest possible value of k that results in a significant coefficient on the k th lag.

(v) If this value of k gives residuals without autocorrelation (using a Breusch-Godfrey Lagrange multiplier tests as described in Chapter 6), use this equation to test for stationarity. If not, increase k until the residuals are free of autocorrelation, and use this equation to test for stationarity.

2. Test the hypothesis $H_0 : \alpha = 0$ against the alternative $H_A : \alpha < 0$.
3. If H_0 is rejected, conclude that the variable is stationary
4. If H_0 is not rejected, go on to estimate a version of (B.4) with $\Delta^2 Y_t$ instead of ΔY_t and ΔY_{t-1} instead of Y_{t-1} . Use $k - 1$ lags. Do not include a trend.
5. Test the hypothesis $H_0 : \alpha = 0$ against the alternative $H_A : \alpha < 0$.
6. If H_0 is rejected, conclude that the variable is I(1)
7. If H_0 is not rejected, continue for another step (and so on).

B.3 Results

We have tested all our variables for stationarity. The results are in Table B.1.

Table B.1: ADF tests

The sample 1975Q4-2009Q1 has been used for all variables in levels

Variable	k	$\hat{\beta}$ (t-value)	ADF-statistic	5 % critical value
sd	4	.0003162 (2.80)	-2.665	-3.446
Δsd	3	-	-1.949	-2.888
$\Delta^2 sd$	2	-	-18.616	-2.888
usd	2	-	-1.254	-2.888
Δusd	1	-	-4.761	-2.888
inc	5	.0005142 (2.10)	-1.544	-3.446
Δinc	4	-	-5.444	-2.888
$inccap$	5	-	-2.187	-2.888
$\Delta inccap$	4	-	-3.905	-2.888

Continued on next page

Table B.1: *Continued from previous page*

$\hat{s}d$	5	.000316 (2.52)	-2.779	-3.446
$\Delta\hat{s}d$	4	-	-2.413	-2.888
$\Delta^2\hat{s}d$	3	-	-14.208	-2.888
$\hat{u}sd$	4	-	-2.277	-2.888
$\Delta\hat{u}sd$	3	-	-3.351	-2.888
hw	5	-	-0.508	-2.888
Δhw	4	-	-3.248	-2.888
$\hat{h}w$	4	-	-2.640	-2.888
$\Delta\hat{h}w$	3	-	-2.336	-2.888
$\Delta^2\hat{h}w$	2	-	-21.826	-2.888
liq	4	-	0.437	-2.888
Δliq	3	-	-3.710	-2.888
$\hat{l}iq$	5	-	-2.769	-2.888
$\Delta\hat{l}iq$	4	-	-4.085	-2.888
$mliq$	5	.0018801 (2.96)	-3.000	-3.446
$\Delta mliq$	4	-	-4.354	-2.888
$\hat{m}liq$	4	.0014008 (3.13)	-3.420	-3.446
$\Delta\hat{m}liq$	3	-	-3.525	-2.888
$illiq$	3	.001163 (2.90)	-2.832	-3.446
$\Delta illiq$	2	-	-4.263	-2.888
$\hat{i}lliq$	4	-	-1.917	-2.888
$\Delta\hat{i}lliq$	3	-	-4.349	-2.888
i	5	-0.0000627 (-2.91)	-2.836	-3.446
Δi	4	-	-3.749	-2.888

Continued on next page

Table B.1: Continued from previous page

r	0	-	-3.017	-2.888
u	4	-	-2.677	-2.888
Δu	3	-	-3.537	-2.888
FTB	6	-0.0000355 (-2.96)	-2.691	-3.446
ΔFTB	5	-	-2.169	-2.888
$\Delta^2 FTB$	4	-	-5.129	-2.888
$incexp$	5	-.0000653 (-3.98)	-5.103	-3.446
$iexp$	0	-	-3.805	-2.888

Based on the tests I interpret usd , \hat{usd} , inc , $inccap$, hw , liq , \hat{liq} , $mliq$, \hat{mliq} , $illq$ and \hat{illq} as I(1), while r , $incexp$ and $iexp$ are interpreted as I(0). Even though both hw and inc are I(1), \hat{hw} shows up as I(2) according to the tests, but graphical inspection (Figure B.1) leads me to accept an I(1) interpretation. It is the same issue for sd and \hat{sd} : the tests support an I(2) interpretation, but looking at Figures B.2-B.3 is "enough" for me to accept them as I(1) variables.

It is harder to handle FTB . By definition it must be an I(0) variable (as it is a proportion), and it would certainly have showed up as that with a sufficient amount of data, but the time it takes for FTB to look like a stationary variable is a lot longer than the 35 years we are looking at. Still, in the short-run it makes sense to think of FTB as a stock variable, making an I(1) or I(2) interpretation more intuitive. If I interpret Figure B.4 with some slack, I choose to treat FTB as an I(1) variable.

Finally, both the unemployment rate and the interest rate show up as I(1), even though I expect these variables to be I(0). For the unemployment rate, the stock-interpretation makes good sense, such that accepting the I(1) label seems sensible. At the same time, one would expect the unemployment rate to act as a stationary variable over a longer sample, but that it is an I(1) variable in our time window seems quite clear from Figure B.5. My plan is in any case to include Δu as a proxy for income uncertainty, making the I(1) interpretation unproblematic. Looking at Figure B.6 it is also hard to argue against the I(1) conclusion for the interest rate. Neither this is a severe problem. The main argument for bringing i into my analysis is to capture the cash-flow effect from changes in the interest rate level; r is included to take care of inter-temporal substitution and the cost of loans. Hence, I might as well include Δi to capture

the cash-flow effect, avoiding the problem of using an $I(1)$ variable in the short-run dynamics (confer the discussion in Chapter 4).

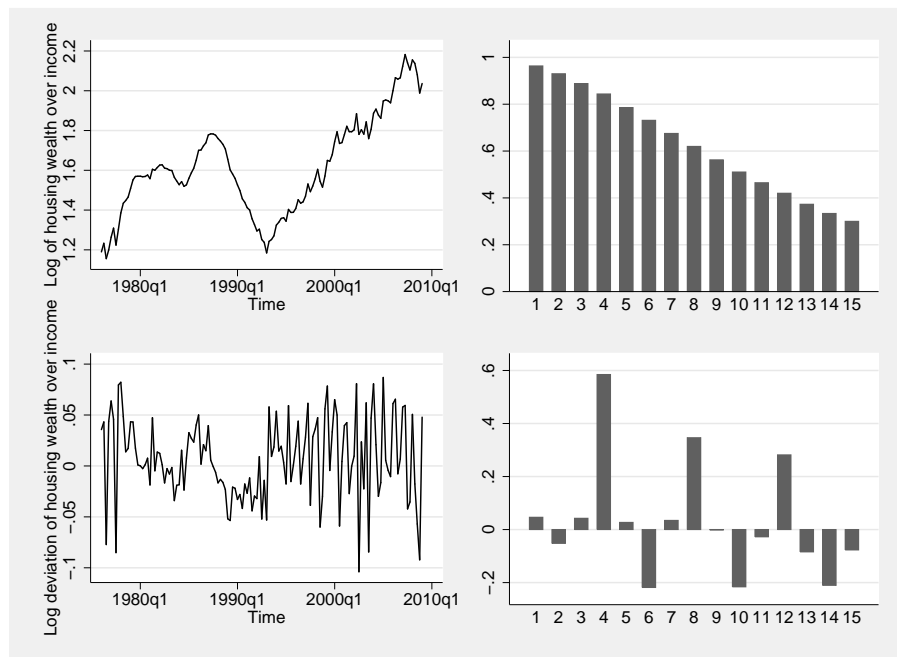


Figure B.1: The development of housing wealth to income and its growth rate (left panels) and the correlograms for the respective series (right panels).

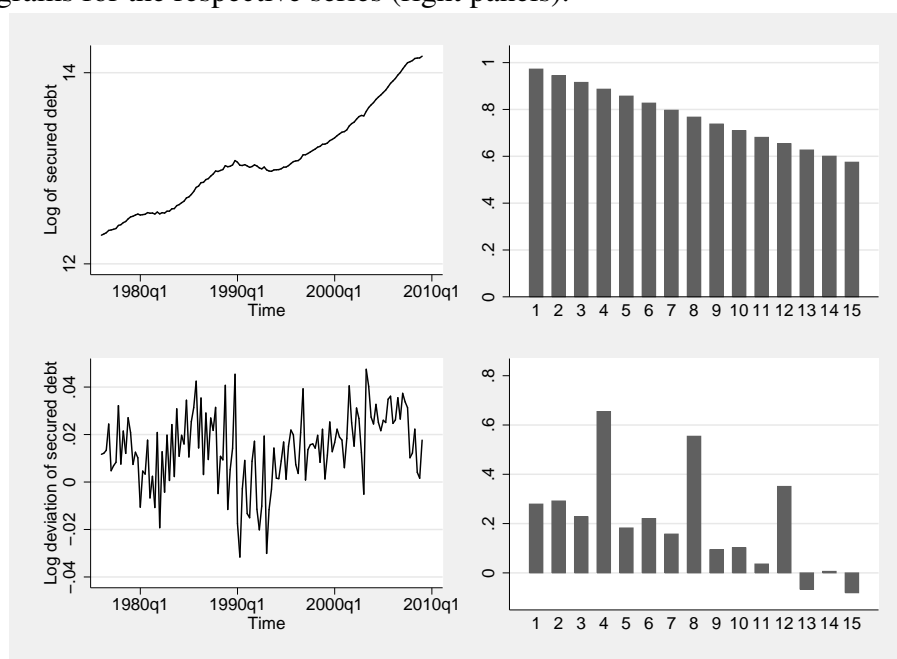


Figure B.2: The development of secured debt and its growth rate (left panels) and the correlograms for the respective series (right panels).

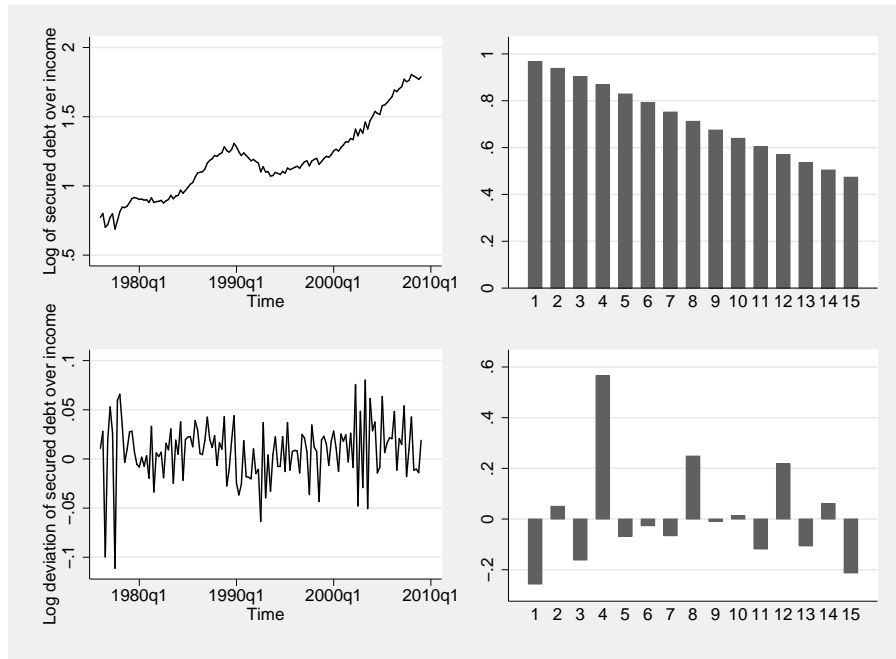


Figure B.3: The development of secured debt over income and its growth rate (left panels) and the correlograms for the respective series (right panels).

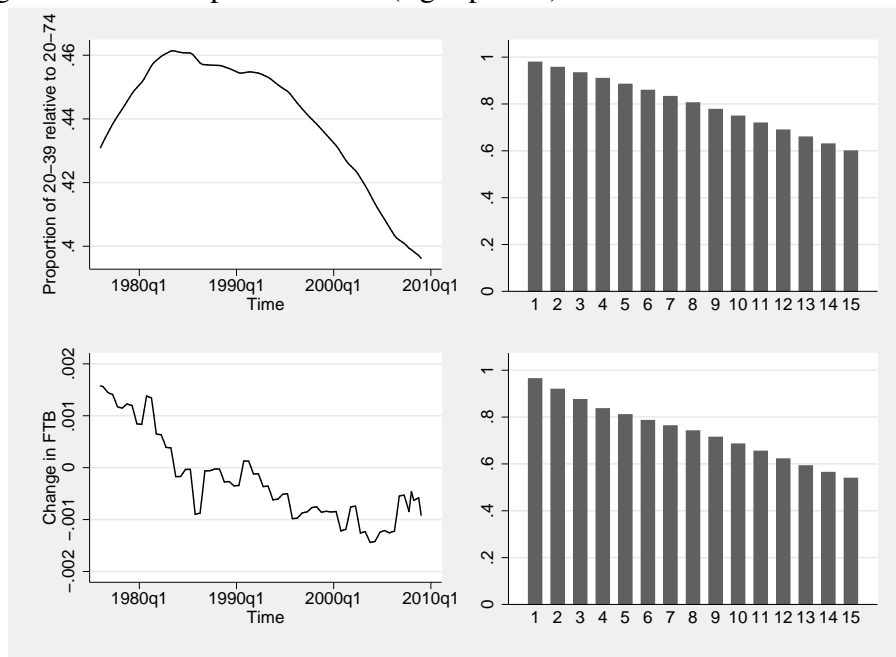


Figure B.4: The development of FTB and its quarterly change (left panels) and the correlograms for the respective series (right panels).

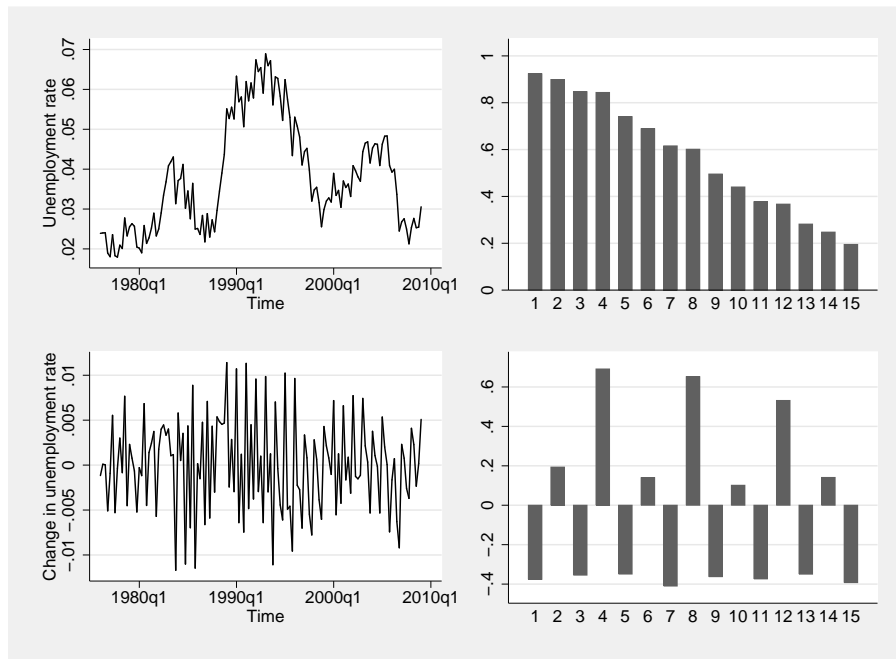


Figure B.5: The development of the unemployment rate and its quarterly change (left panels) and the correlograms for the respective series (right panels).

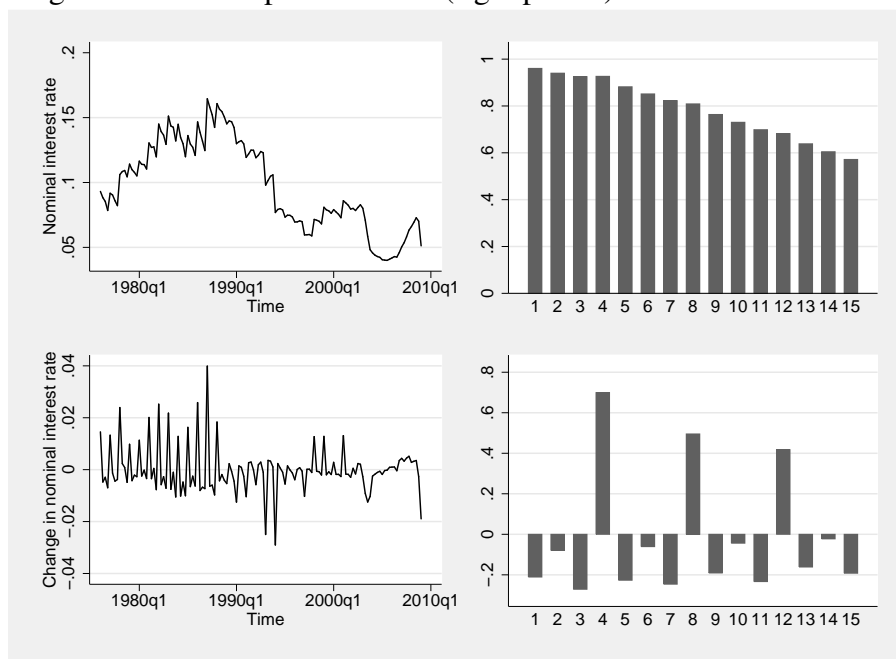


Figure B.6: The development of the interest rate and its quarterly change (left panels) and the correlograms for the respective series (right panels).

C Derivations

Before deriving any results, let us define what is called *ML equations* in Stata, which is a way of simplifying the syntax in a program. We define:

$$\theta_{1t} = \Phi_1' z_t \quad (\text{C.1})$$

$$\theta_{2t} = \Phi_2' z_t \quad (\text{C.2})$$

$$\theta_{3t} = B_2' c_{t-1} \quad (\text{C.3})$$

$$\theta_{4t} = \tilde{\gamma} \quad (\text{C.4})$$

$$\theta_{5t} = \sigma_{11} \quad (\text{C.5})$$

$$\theta_{6t} = \sigma_{12} \quad (\text{C.6})$$

$$\theta_{7t} = \sigma_{22} \quad (\text{C.7})$$

The reason for defining these θ 's is to simplify for instance the coding of the gradient. In a typical program you will only have to code the derivatives with respect to θ_{it} , and special Stata commands are developed to multiply with the correct variable vector and sum over all t . The usefulness of this syntax therefore becomes more clear as soon as you try to program these results in Stata.

C.1 Gradient of the log likelihood function

I want the vector containing the derivatives of the likelihood function in (5.7). Using our ML-equations notation we see that the gradient is given as:

$$g = \sum_t \begin{pmatrix} g_{1t} z_t \\ g_{2t} z_t \\ g_{3t} c_t \\ g_{4t} \\ g_{5t} \\ g_{6t} \\ g_{7t} \end{pmatrix} \quad (\text{C.8})$$

where $g_{it} = \frac{\partial \ln L_t}{\partial \theta_{it}}$ for $i = 1, 2, \dots, 7$. Hence the search for the gradient is reduced to the job of just finding these 7 derivatives.

For the first 4 elements we find:

$$\begin{aligned}
g_{1t} &= \sigma^{11} r_{1t} + \sigma^{12} r_{2t} \\
g_{2t} &= \sigma^{12} r_{1t} + \sigma^{22} r_{2t} \\
g_{3t} &= g_{1t} + \theta_{4t} g_{2t} \\
g_{4t} &= \theta_{3t} g_{2t}
\end{aligned}$$

where σ_{ij} is the (i, j) th element of Ω and σ^{ij} is the (i, j) th element of Ω^{-1} ($i, j = 1, 2$). In our two-equation case we have:

$$\begin{aligned}
\frac{\partial \sigma^{aa}}{\partial \sigma_{aa}} &= -\sigma^{aa} \sigma^{aa} \\
\frac{\partial \sigma^{aa}}{\partial \sigma_{12}} &= -2\sigma^{aa} \sigma^{12} \\
\frac{\partial \sigma^{12}}{\partial \sigma_{aa}} &= -\sigma^{aa} \sigma^{12} \\
\frac{\partial \sigma^{aa}}{\partial \sigma_{bb}} &= -\sigma^{12} \sigma^{12} \\
\frac{\partial \sigma^{12}}{\partial \sigma_{12}} &= -\sigma^{11} \sigma^{22} - \sigma^{12} \sigma^{12}
\end{aligned}$$

for $a, b = 1, 2$, making the last three elements of the gradient defined as:

$$\begin{aligned}
g_{5t} &= \frac{1}{2} ((\sigma^{11} r_{1t})^2 + (\sigma^{12} r_{2t})^2 + 2\sigma^{11} \sigma^{12} r_{1t} r_{2t} - \sigma^{11}) \\
g_{6t} &= \sigma^{11} \sigma^{12} r_{1t}^2 + \sigma^{22} \sigma^{12} r_{2t}^2 + (\sigma^{11} \sigma^{22} + \sigma^{12} \sigma^{12}) r_{1t} r_{2t} - \sigma^{12} \\
g_{7t} &= \frac{1}{2} ((\sigma^{22} r_{2t})^2 + (\sigma^{12} r_{1t})^2 + 2\sigma^{22} \sigma^{12} r_{1t} r_{2t} - \sigma^{22})
\end{aligned}$$

Insert for all the g_{it} elements into (C.8) to get:

$$g = \sum_t \begin{pmatrix} (\sigma^{11} r_{1t} + \sigma^{12} r_{2t}) z_t \\ (\sigma^{12} r_{1t} + \sigma^{22} r_{2t}) z_t \\ (\sigma^{11} r_{1t} + \sigma^{12} r_{2t} + \theta_{4t} (\sigma^{12} r_{1t} + \sigma^{22} r_{2t})) c_t \\ \theta_{3t} (\sigma^{12} r_{1t} + \sigma^{22} r_{2t}) \\ \frac{1}{2} ((\sigma^{11} r_{1t})^2 + (\sigma^{12} r_{2t})^2 + 2\sigma^{11} \sigma^{12} r_{1t} r_{2t} - \sigma^{11}) \\ \sigma^{11} \sigma^{12} r_{1t}^2 + \sigma^{22} \sigma^{12} r_{2t}^2 + (\sigma^{11} \sigma^{22} + \sigma^{12} \sigma^{12}) r_{1t} r_{2t} - \sigma^{12} \\ \frac{1}{2} ((\sigma^{22} r_{2t})^2 + (\sigma^{12} r_{1t})^2 + 2\sigma^{22} \sigma^{12} r_{1t} r_{2t} - \sigma^{22}) \end{pmatrix} \quad (C.9)$$

C.2 Hessian of the likelihood function

We need the matrix containing all the cross derivatives of the likelihood function in (5.7). I find that the Hessian is given as:

$$H = \sum_t \begin{pmatrix} z_t \frac{\partial g_{1t}}{\partial \Phi_1'} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ z_t \frac{\partial g_{1t}}{\partial \Phi_2'} & z_t \frac{\partial g_{2t}}{\partial \Phi_2'} & \cdot & \cdot & \cdot & \cdot & \cdot \\ z_t \frac{\partial g_{1t}}{\partial B_2'} & z_t \frac{\partial g_{2t}}{\partial B_2'} & c_t \frac{\partial g_{3t}}{\partial B_2'} & \cdot & \cdot & \cdot & \cdot \\ z_t \frac{\partial g_{1t}}{\partial \tilde{\gamma}} & z_t \frac{\partial g_{2t}}{\partial \tilde{\gamma}} & c_t \frac{\partial g_{3t}}{\partial \tilde{\gamma}} & \frac{\partial g_{4t}}{\partial \tilde{\gamma}} & \cdot & \cdot & \cdot \\ z_t \frac{\partial g_{1t}}{\partial \sigma_{11}} & z_t \frac{\partial g_{2t}}{\partial \sigma_{11}} & c_t \frac{\partial g_{3t}}{\partial \sigma_{11}} & \frac{\partial g_{4t}}{\partial \sigma_{11}} & \frac{\partial g_{5t}}{\partial \sigma_{11}} & \cdot & \cdot \\ z_t \frac{\partial g_{1t}}{\partial \sigma_{12}} & z_t \frac{\partial g_{2t}}{\partial \sigma_{12}} & c_t \frac{\partial g_{3t}}{\partial \sigma_{12}} & \frac{\partial g_{4t}}{\partial \sigma_{12}} & \frac{\partial g_{5t}}{\partial \sigma_{12}} & \frac{\partial g_{6t}}{\partial \sigma_{12}} & \cdot \\ z_t \frac{\partial g_{1t}}{\partial \sigma_{22}} & z_t \frac{\partial g_{2t}}{\partial \sigma_{22}} & c_t \frac{\partial g_{3t}}{\partial \sigma_{22}} & \frac{\partial g_{4t}}{\partial \sigma_{22}} & \frac{\partial g_{5t}}{\partial \sigma_{22}} & \frac{\partial g_{6t}}{\partial \sigma_{22}} & \frac{\partial g_{7t}}{\partial \sigma_{22}} \end{pmatrix} \quad (C.10)$$

where $\frac{\partial g_{it}}{\partial \lambda'}$ is a vector of the same dimension as λ containing the derivative of g_{it} with respect to all the elements of λ . We must find all these derivatives:

$$\begin{aligned} \frac{\partial g_{1t}}{\partial \Phi_1'} &= -\sigma^{11} z_t \\ \frac{\partial g_{1t}}{\partial \Phi_2'} &= -\sigma^{12} z_t \\ \frac{\partial g_{1t}}{\partial B_2'} &= -(\sigma^{11} + \theta_{4t} \sigma^{12}) c_t \\ \frac{\partial g_{1t}}{\partial \tilde{\gamma}} &= -\theta_{3t} \sigma^{12} \\ \frac{\partial g_{1t}}{\partial \sigma_{11}} &= -\sigma^{11} g_{1t} \\ \frac{\partial g_{1t}}{\partial \sigma_{12}} &= -\sigma^{12} g_{1t} - \sigma^{11} g_{2t} \\ \frac{\partial g_{1t}}{\partial \sigma_{22}} &= -\sigma^{12} g_{2t} \\ \\ \frac{\partial g_{2t}}{\partial \Phi_2'} &= -\sigma^{22} z_t \\ \frac{\partial g_{2t}}{\partial B_2'} &= -(\theta_{4t} \sigma^{22} + \sigma^{12}) c_t \\ \frac{\partial g_{2t}}{\partial \tilde{\gamma}} &= -\theta_{3t} \sigma^{22} \end{aligned}$$

$$\begin{aligned}\frac{\partial g_{2t}}{\partial \sigma_{11}} &= -\sigma^{12} g_{1t} \\ \frac{\partial g_{2t}}{\partial \sigma_{12}} &= -\sigma^{22} g_{1t} - \sigma^{12} g_{2t} \\ \frac{\partial g_{2t}}{\partial \sigma_{22}} &= -\sigma^{22} g_{2t}\end{aligned}$$

$$\begin{aligned}\frac{\partial g_{3t}}{\partial B'_2} &= -(\sigma^{11} + 2\theta_{4t}\sigma^{12} + \theta_{4t}^2\sigma^{22})c_t \\ \frac{\partial g_{3t}}{\partial \tilde{\gamma}} &= -\theta_{3t}\sigma^{12} - \theta_{3t}\theta_{4t}\sigma^{22} + g_{2t} \\ \frac{\partial g_{3t}}{\partial \sigma_{11}} &= -(\sigma^{11} + \theta_{4t}\sigma^{12})g_{1t} \\ \frac{\partial g_{3t}}{\partial \sigma_{12}} &= -[(\sigma^{12} + \theta_{4t}\sigma^{22})g_{1t} + (\sigma^{11} + \theta_{4t}\sigma^{12})g_{2t}] \\ \frac{\partial g_{3t}}{\partial \sigma_{22}} &= -(\sigma^{12} + \theta_{4t}\sigma^{22})g_{2t} \\ \frac{\partial g_{4t}}{\partial \tilde{\gamma}} &= -\theta_{3t}^2\sigma^{22} \\ \frac{\partial g_{4t}}{\partial \sigma_{11}} &= -\theta_{3t}\sigma^{12}g_{1t} \\ \frac{\partial g_{4t}}{\partial \sigma_{12}} &= -\theta_{3t}(\sigma^{22}g_{1t} + \sigma^{12}g_{2t}) \\ \frac{\partial g_{4t}}{\partial \sigma_{22}} &= -\theta_{3t}\sigma^{22}g_{2t}\end{aligned}$$

$$\begin{aligned}\frac{\partial g_{5t}}{\partial \sigma_{11}} &= -2\sigma^{11}g_{5t} - \frac{1}{2}\sigma^{11}\sigma^{11} \\ \frac{\partial g_{5t}}{\partial \sigma_{12}} &= -2\sigma^{12}g_{5t} - \sigma^{11}g_{6t} - \sigma^{11}\sigma^{12} \\ \frac{\partial g_{5t}}{\partial \sigma_{22}} &= -2\sigma^{12}g_{6t} - \frac{1}{2}\sigma^{12}\sigma^{12} \\ \frac{\partial g_{6t}}{\partial \sigma_{12}} &= -2\sigma^{12}g_{6t} - \sigma^{11}g_{7t} - \sigma^{22}g_{5t} - (\sigma^{11}\sigma^{22} + \sigma^{12}\sigma^{12}) \\ \frac{\partial g_{6t}}{\partial \sigma_{22}} &= -\sigma^{22}g_{6t} - \sigma^{12}g_{7t} - \sigma^{22}\sigma^{12}\end{aligned}$$

$$\frac{\partial g_{7t}}{\partial \sigma_{22}} = -2\sigma^{22}g_{7t} - \frac{1}{2}\sigma^{22}\sigma^{22}$$

When we insert all these expressions in (C.10) we get

$$H = \sum_t H_t \quad (\text{C.11})$$

where we have:

$$H_t = \begin{pmatrix} h_{11t}z_t z_t' & . & . & . & . & . & . \\ h_{21t}z_t z_t' & h_{22t}z_t z_t' & . & . & . & . & . \\ h_{31t}c_t z_t' & h_{32t}c_t z_t' & h_{33t}c_t' c_t & . & . & . & . \\ h_{41t}z_t & h_{42t}z_t & h_{43t}c_t & h_{44t} & . & . & . \\ h_{51t}z_t & h_{52t}z_t & h_{53t}c_t & h_{54t} & h_{55t} & . & . \\ h_{61t}z_t & h_{62t}z_t & h_{63t}c_t & h_{64t} & h_{65t} & h_{66t} & . \\ h_{71t}z_t & h_{72t}z_t & h_{73t}c_t & h_{74t} & h_{75t} & h_{76t} & h_{77t} \end{pmatrix} \quad (\text{C.12})$$

with elements h_{ijt} defined as:

$$h_{11t} = -\sigma^{11} \quad (\text{C.13})$$

$$h_{21t} = -\sigma^{12} \quad (\text{C.14})$$

$$h_{31t} = -\sigma^{11} - \theta_{4t}\sigma^{12} \quad (\text{C.15})$$

$$h_{41t} = -\theta_{3t}\sigma^{12} \quad (\text{C.16})$$

$$h_{51t} = -\sigma^{11}g_{1t} \quad (\text{C.17})$$

$$h_{61t} = -\sigma^{12}g_{1t} - \sigma^{11}g_{2t} \quad (\text{C.18})$$

$$h_{71t} = -\sigma^{12}g_{2t} \quad (\text{C.19})$$

$$h_{22t} = -\sigma^{22} \quad (\text{C.20})$$

$$h_{32t} = -\theta_{4t}\sigma^{22} - \sigma^{12} \quad (\text{C.21})$$

$$h_{42t} = -\theta_{3t}\sigma^{22} \quad (\text{C.22})$$

$$h_{52t} = -\sigma^{12}g_{1t} \quad (\text{C.23})$$

$$h_{62t} = -\sigma^{22}g_{1t} - \sigma^{12}g_{2t} \quad (\text{C.24})$$

$$h_{72t} = -\sigma^{22}g_{2t} \quad (\text{C.25})$$

$$h_{33t} = -\sigma^{11} - 2\theta_{4t}\sigma^{12} - \theta_{4t}^2\sigma^{22} \quad (\text{C.26})$$

$$h_{43t} = -\theta_{3t}\sigma^{12} - \theta_{3t}\theta_{4t}\sigma^{22} + g_{2t} \quad (\text{C.27})$$

$$h_{53t} = -(\sigma^{11} + \theta_{4t}\sigma^{12})g_{1t} \quad (\text{C.28})$$

$$h_{63t} = -[(\sigma^{12} + \theta_{4t}\sigma^{22})g_{1t} + (\sigma^{11} + \theta_{4t}\sigma^{12})g_{2t}] \quad (\text{C.29})$$

$$h_{73t} = -(\sigma^{12} + \theta_{4t}\sigma^{22})g_{2t} \quad (\text{C.30})$$

$$h_{44t} = -\theta_{3t}^2 \sigma^{22} \quad (C.31)$$

$$h_{54t} = -\theta_{3t} \sigma^{12} g_{1t} \quad (C.32)$$

$$h_{64t} = -\theta_{3t} (\sigma^{22} g_{1t} + \sigma^{12} g_{2t}) \quad (C.33)$$

$$h_{74t} = -\theta_{3t} \sigma^{22} g_{2t} \quad (C.34)$$

$$h_{55t} = -2\sigma^{11} g_{5t} - \frac{1}{2} \sigma^{11} \sigma^{11} \quad (C.35)$$

$$h_{65t} = -2\sigma^{12} g_{5t} - \sigma^{11} g_{6t} - \sigma^{11} \sigma^{12} \quad (C.36)$$

$$h_{75t} = -2\sigma^{12} g_{6t} - \frac{1}{2} \sigma^{12} \sigma^{12} \quad (C.37)$$

$$h_{66t} = -2\sigma^{12} g_{6t} - \sigma^{11} g_{7t} - \sigma^{22} g_{5t} - (\sigma^{11} \sigma^{22} + \sigma^{12} \sigma^{12}) \quad (C.38)$$

$$h_{76t} = -\sigma^{22} g_{6t} - \sigma^{12} g_{7t} - \sigma^{22} \sigma^{12} \quad (C.39)$$

$$h_{77t} = -2\sigma^{22} g_{7t} - \frac{1}{2} \sigma^{22} \sigma^{22} \quad (C.40)$$

C.3 Gradient of the concentrated likelihood

We need the gradient of the likelihood function in (5.11). Similar to the previous case, the gradient is:

$$g_\beta = \sum_t \begin{pmatrix} g_{\beta 1t} z_t \\ g_{\beta 2t} z_t \\ g_{\beta 3t} c_t \\ g_{\beta 4t} \end{pmatrix} \quad (C.41)$$

where $g_{\beta it} = \frac{\partial \ln L_\beta}{\partial \theta_{it}}$. Let us find these elements.

First, I denote the (a, b) th element of W as w_{ab} , and the (a, b) th element of the inverse of W as w^{ab} . w_{ab} is thus defined as:

$$w_{ab} = \frac{1}{T} \sum_t r_{at} r_{bt}$$

for $a, b = 1, 2$. I differentiate $|W|$ with respect to θ_t , the vector of all θ_i 's at time t :

$$\frac{\partial |W|}{\partial \theta_t} = \frac{\partial w_{11}}{\partial \theta_t} w_{22} + \frac{\partial w_{22}}{\partial \theta_t} w_{11} - 2 \frac{\partial w_{12}}{\partial \theta_t} w_{12}$$

If we now consider the whole likelihood function we see that:

$$\frac{\partial \ln L_\beta}{\partial \theta_t} = -\frac{T}{2} \frac{1}{|W|} \frac{\partial |W|}{\partial \theta_t}$$

making

$$\frac{\partial \ln L_\beta}{\partial \theta_t} = -\frac{T}{2} \left(\frac{\partial w_{11}}{\partial \theta_t} w^{11} + \frac{\partial w_{22}}{\partial \theta_t} w^{22} + 2 \frac{\partial w_{12}}{\partial \theta_t} w^{12} \right)$$

The various derivatives are given as:

$$\begin{aligned}\frac{\partial w_{11}}{\partial \theta_t} &= -\frac{2}{T} r_{1t} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \\ \frac{\partial w_{22}}{\partial \theta_t} &= -\frac{2}{T} r_{2t} \begin{pmatrix} 0 \\ 1 \\ \theta_{4t} \\ \theta_{3t} \end{pmatrix} \\ \frac{\partial w_{12}}{\partial \theta_t} &= -\frac{1}{T} r_{1t} \begin{pmatrix} 0 \\ 1 \\ \theta_{4t} \\ \theta_{3t} \end{pmatrix} - \frac{1}{T} r_{2t} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}\end{aligned}$$

Combining these results we get

$$g_\beta = \sum_t \begin{pmatrix} (w^{11} r_{1t} + w^{12} r_{2t}) z_t \\ (w^{12} r_{1t} + w^{22} r_{2t}) z_t \\ ((w^{11} + \theta_{4t} w^{12}) r_{1t} + (\theta_{4t} w^{22} + w^{12}) r_{2t}) c_t \\ \theta_{3t} w^{12} r_{1t} + \theta_{3t} w^{22} r_{2t} \end{pmatrix} \quad (\text{C.42})$$

C.4 Hessian of the concentrated likelihood

We want the matrix containing all the cross derivatives of the concentrated likelihood function defined in (5.11). The Hessian takes the form of a smaller version of (C.10):

$$H_\beta = \sum_t \begin{pmatrix} z_t \frac{\partial g_{\beta 1t}}{\partial \Phi_1'} & \cdot & \cdot & \cdot \\ z_t \frac{\partial g_{\beta 1t}}{\partial \Phi_2'} & z_t \frac{\partial g_{\beta 2t}}{\partial \Phi_2'} & \cdot & \cdot \\ z_t \frac{\partial g_{\beta 1t}}{\partial B_2'} & z_t \frac{\partial g_{\beta 2t}}{\partial B_2'} & c_t \frac{\partial g_{\beta 3t}}{\partial B_2'} & \cdot \\ z_t \frac{\partial g_{\beta 1t}}{\partial \tilde{\gamma}} & z_t \frac{\partial g_{\beta 2t}}{\partial \tilde{\gamma}} & c_t \frac{\partial g_{\beta 3t}}{\partial \tilde{\gamma}} & \frac{\partial g_{\beta 4t}}{\partial \tilde{\gamma}} \end{pmatrix}$$

where $\frac{\partial g_{\beta it}}{\partial \lambda'}$ is a vector of the same dimension as λ containing the derivative of $g_{\beta it}$ with respect to all the elements of λ . In contrast to the case with the standard likelihood-function, these derivatives will take a quite complicated form, but below we will show an approximation we

will use. For $g_{\beta 1t}$ we find:

$$\begin{aligned}\frac{\partial g_{\beta 1t}}{\partial \Phi'_1} &= -w^{11}z_t + \frac{\partial w^{11}}{\partial \Phi'_1}r_{1t} + \frac{\partial w^{12}}{\partial \Phi'_1}r_{2t} \\ \frac{\partial g_{\beta 1t}}{\partial \Phi'_2} &= -w^{12}z_t + \frac{\partial w^{11}}{\partial \Phi'_2}r_{1t} + \frac{\partial w^{12}}{\partial \Phi'_2}r_{2t} \\ \frac{\partial g_{\beta 1t}}{\partial B'_2} &= -(w^{11} + w^{12}\tilde{\gamma})c_t + \frac{\partial w^{11}}{\partial B'_2}r_{1t} + \frac{\partial w^{12}}{\partial B'_2}r_{2t} \\ \frac{\partial g_{\beta 1t}}{\partial \tilde{\gamma}} &= -w^{12}\theta_{3t} + \frac{\partial w^{11}}{\partial \tilde{\gamma}}r_{1t} + \frac{\partial w^{12}}{\partial \tilde{\gamma}}r_{2t}\end{aligned}$$

where now $\frac{\partial w^{ab}}{\partial \lambda'}$ is a vector of the same dimension as λ containing the derivative of g^{ab} with respect to all the elements of λ . A similar pattern follows for $g_{\beta 2t}$:

$$\begin{aligned}\frac{\partial g_{\beta 2t}}{\partial \Phi'_2} &= -w^{22}z_t + \frac{\partial w^{22}}{\partial \Phi'_2}r_{2t} + \frac{\partial w^{12}}{\partial \Phi'_2}r_{1t} \\ \frac{\partial g_{\beta 2t}}{\partial B'_2} &= -(w^{12} + w^{22}\tilde{\gamma})c_t + \frac{\partial w^{22}}{\partial B'_2}r_{2t} + \frac{\partial w^{12}}{\partial B'_2}r_{1t} \\ \frac{\partial g_{\beta 2t}}{\partial \tilde{\gamma}} &= -w^{22}\theta_{3t} + \frac{\partial w^{22}}{\partial \tilde{\gamma}}r_{2t} + \frac{\partial w^{12}}{\partial \tilde{\gamma}}r_{1t}\end{aligned}$$

Since $g_{\beta 3t}$ and $g_{\beta 4t}$ are functions of the two first we get:

$$\begin{aligned}\frac{\partial g_{\beta 3t}}{\partial B'_2} &= \frac{\partial g_{\beta 1t}}{\partial B'_2} + \tilde{\gamma} \frac{\partial g_{\beta 2t}}{\partial B'_2} \\ \frac{\partial g_{\beta 3t}}{\partial \tilde{\gamma}} &= \frac{\partial g_{\beta 1t}}{\partial \tilde{\gamma}} + \tilde{\gamma} \frac{\partial g_{\beta 2t}}{\partial \tilde{\gamma}} + g_{\beta 2t} \\ \frac{\partial g_{\beta 4t}}{\partial \tilde{\gamma}} &= \theta_{3t} \frac{\partial g_{\beta 2t}}{\partial \tilde{\gamma}}\end{aligned}$$

I will not calculate the exact Hessian. Instead I will use an approximation, following Gould, Pitblado, and Sribney (2003) which decides to ignore all the $\frac{\partial w^{ab}}{\partial \lambda'}$ terms as these go to zero at the same rate as the elements of the gradient. What I then end up with is

$$H_{\beta} = \sum_t H_{\beta t} \tag{C.43}$$

where

$$H_{\beta t} = \begin{pmatrix} h_{\beta 11t}z_t z'_t & \cdot & \cdot & \cdot \\ h_{\beta 21t}z_t z'_t & h_{\beta 22t}z_t z'_t & \cdot & \cdot \\ h_{\beta 31t}c_t z'_t & h_{\beta 32t}c_t z'_t & h_{\beta 33t}c_t c'_t & \cdot \\ h_{\beta 41t}z_t & h_{\beta 42t}z_t & h_{\beta 43t}c_t & h_{\beta 44t} \end{pmatrix}$$

with elements $h_{\beta ij}$ given as:

$$h_{\beta 11} = -w^{11} \quad (\text{C.44})$$

$$h_{\beta 21t} = -w^{12} \quad (\text{C.45})$$

$$h_{\beta 31t} = -(w^{11} + w^{12}\theta_{4t}) \quad (\text{C.46})$$

$$h_{\beta 41t} = -w^{12}\theta_{3t} \quad (\text{C.47})$$

$$h_{\beta 22t} = -w^{22} \quad (\text{C.48})$$

$$h_{\beta 32t} = -(w^{12} + w^{22}\theta_{4t}) \quad (\text{C.49})$$

$$h_{\beta 42t} = -w^{22}\theta_{3t} \quad (\text{C.50})$$

$$h_{\beta 33t} = -(w^{11} + 2w^{12}\theta_{4t} + w^{22}\theta_{4t}^2) \quad (\text{C.51})$$

$$h_{\beta 43t} = -w^{22}(\theta_{3t}\theta_{4t} - r_{2t}) - w^{12}(\theta_{3t} - r_{1t}) \quad (\text{C.52})$$

$$h_{\beta 44t} = -w^{22}\theta_{3t}^2 \quad (\text{C.53})$$

C.5 Gradient of the gamma-concentrated likelihood

When deriving the gradient of the likelihood function in (5.14) we will attack the problem more directly than what we did in the previous two cases. The gradient is:

$$g_{\beta\gamma} = \begin{pmatrix} \frac{\partial \ln L_{\beta\gamma}}{\partial \Phi'_1} \\ \frac{\partial \ln L_{\beta\gamma}}{\partial \Phi'_2} \\ \frac{\partial \ln L_{\beta\gamma}}{\partial B'_2} \end{pmatrix} \quad (\text{C.54})$$

with

$$\frac{\partial \ln L_{\beta\gamma}}{\partial \lambda'} = -\frac{T}{2} \frac{1}{|W|} \frac{\partial |W|}{\partial \lambda'} \quad (\text{C.55})$$

Let us now denote the derivatives of A , B , and $|W|$ with respect to the transpose of the coefficient vector of ML equation i as A_i , B_i and W_i , respectively. These derivatives will be functions of the terms s_{abi} , which are the derivatives of s_{ab} with respect to the transpose of the coefficient vector of equation i . We find that:

$$W_i = \frac{1}{T^2} (s_{11i}s_{22} + s_{22i}s_{11} - 2s_{21i}s_{21} - 2\tilde{\gamma}A_i + \tilde{\gamma}^2B_i) \quad (\text{C.56})$$

$$A_i = s_{11i}s_{32} + s_{32i}s_{11} - s_{31i}s_{21} - s_{21i}s_{31} \quad (\text{C.57})$$

$$B_i = s_{11i}s_{33} + s_{33i}s_{11} - 2s_{31i}s_{31} \quad (\text{C.58})$$

for $i = 1, 2, 3$ and

$$\begin{aligned}
s_{111} &= -2 \sum_t r_{1t} z_t \\
s_{112} &= 0_{k_z} \\
s_{113} &= -2 \sum_t r_{1t} c_t \\
s_{211} &= - \sum_t \hat{r}_{2t} z_t \\
s_{212} &= - \sum_t r_{1t} z_t \\
s_{213} &= - \sum_t \hat{r}_{2t} c_t \\
s_{221} &= 0_{k_z} \\
s_{222} &= -2 \sum_t \hat{r}_{2t} z_t \\
s_{223} &= 0_{k_c} \\
s_{311} &= - \sum_t \theta_{3t} z_t \\
s_{312} &= 0_{k_z} \\
s_{313} &= \sum_t (r_{1t} - \theta_{3t}) c_t \\
s_{321} &= 0_{k_z} \\
s_{322} &= - \sum_t \theta_{3t} z_t \\
s_{323} &= \sum_t \hat{r}_{2t} c_t \\
s_{331} &= 0_{k_z} \\
s_{332} &= 0_{k_z} \\
s_{333} &= 2 \sum_t \theta_{3t} c_t
\end{aligned}$$

Plug these results into (C.54) to get the full gradient.

C.6 Hessian of the gamma-concentrated likelihood

The Hessian will take the form:

$$H_{\beta\gamma} = \begin{pmatrix} \frac{\partial^2 \ln L_{\beta\gamma}}{\partial \Phi_1 \partial \Phi_1'} & \cdot & \cdot \\ \frac{\partial^2 \ln L_{\beta\gamma}}{\partial \Phi_2 \partial \Phi_1'} & \frac{\partial^2 \ln L_{\beta\gamma}}{\partial \Phi_2 \partial \Phi_2'} & \cdot \\ \frac{\partial^2 \ln L_{\beta\gamma}}{\partial B_2 \partial \Phi_1'} & \frac{\partial^2 \ln L_{\beta\gamma}}{\partial B_2 \partial \Phi_2'} & \frac{\partial^2 \ln L_{\beta\gamma}}{\partial B_2 \partial B_2'} \end{pmatrix} \quad (\text{C.59})$$

The various sub-matrices in the Hessian are given as:

$$\frac{\partial^2 \ln L_{\beta\gamma}}{\partial \lambda_i \partial \lambda'_j} = -\frac{T}{2} \frac{1}{|W|} \left(\frac{\partial^2 |W|}{\partial \lambda_i \partial \lambda'_j} - \frac{1}{|W|} \frac{\partial |W|}{\partial \lambda'_i} \frac{\partial |W|}{\partial \lambda_j} \right) \quad (\text{C.60})$$

Let now W_{ij} , A_{ij} and B_{ij} denote the derivatives of W_i , A_i and B_i with respect to the coefficient vector of equation j , while G_i is the derivative of $\tilde{\gamma}_{ML}$ with respect to the transpose of the coefficient vector of equation i . With formulas for these, the Hessian is also defined. We have:

$$\begin{aligned} W_{ij} = & \frac{1}{T^2} (s_{11ij}s_{22} + s_{11i}s'_{22j} + s_{22ij}s_{11} + s_{22i}s'_{11j} \\ & - 2s_{21ij}s_{21} - 2s_{21i}s'_{21j} \\ & - 2(A_i - \tilde{\gamma}B_i)G'_j - 2\tilde{\gamma}A_{ij} + \tilde{\gamma}^2 B_{ij}) \end{aligned} \quad (\text{C.61})$$

$$G_i = (A_i - \tilde{\gamma}B_i)/B \quad (\text{C.62})$$

$$\begin{aligned} A_{ij} = & s_{11ij}s_{32} + s_{11i}s'_{32j} + s_{32ij}s_{11} + s_{32i}s'_{11j} \\ & - s_{31ij}s_{21} - s_{31i}s'_{21j} - s_{21ij}s_{31} - s_{21i}s'_{31j} \end{aligned} \quad (\text{C.63})$$

$$\begin{aligned} B_{ij} = & s_{11ij}s_{33} + s_{11i}s'_{33j} + s_{33ij}s_{11} + s_{33i}s'_{11j} \\ & - 2s_{31ij}s_{31} - 2s_{31i}s'_{31j} \end{aligned} \quad (\text{C.64})$$

for $i, j = 1, 2, 3$ and where s_{ijkl} is the derivative of s_{ijk} with respect to the coefficient vector of equation l (explicit formulas for these are superfluous) for $k, l = 1, 2, 3$.